Math 347 **Homework 4** **Due Mon., Oct. 1, 2001**

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1. – (ungraded) 12.1.

2. – (ungraded) 12.3.

3. – 12.4.

4. – 12.6.

5. – 12.8.

6. – 13.4.

7. – Find a sequence (s_n) with the property that $\sup s_n = 4$, $\limsup s_n = 3$, $\liminf s_n = 2$ and $\inf s_n = 1$.

8. – Suppose (s_n) is a bounded, but not convergent, sequence and $s_n > 0$.
a. If $t_n \to t \neq 0$, prove that $(s_n t_n)$ is not convergent.
b. Let $s_n = 2 + (-1)^n$ (which satisfies the criteria of this problem.) Find two bounded, but not convergent, sequences (u_n) and (v_n) so that $u_n \geq 1$ and $v_n \geq 1$ and $(s_n u_n)$ is convergent, but $(s_n v_n)$ is not convergent.

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9. – 12.12.

10. – Recall that the *Whitman sequence* (W_n) ("I contain multitudes") consists of the ordered blocks of all decimal expansions with n digits, written in increasing order, as n runs from 1 to ∞. That is, the sequence is

$$0, .1, \dots, .9, 1, 0, .01, \dots, .99.1, 0, .001, \dots, .999, 1, 0, .0001, \dots, .9999, 1, 0, .00001, \dots$$

We've seen that every $x \in [0, 1]$ is a subsequential limit from the Whitman sequence, upon taking the sequence of consecutive decimal approximations. (For example $\frac{\pi}{10}$ is the limit of the subsequence $.3, .31, .314, .3141, \dots$.)

In this problem, your job is to modify the Whitman sequence by creating a sequence (z_n), **which contains $(\mathbf{W(n)})$ as a subsequence**, and which has the property that its subsequential limits are $\mathbf{R}$. Note: I think Example 3, p.65 is **not** the right approach.