Math 347 **Homework 5** **Due Mon., Oct. 8, 2001**

---

1. – 10.8 (Hint: first prove that $s_n \geq \sigma_n$, and then write σ_n as a linear combination of σ_{n-1} and s_n.)

2. – 12.9 (ungraded).

3. – 12.13 (ungraded).

4. – 13.10a,b.

5. – 13.12.

6. – 14.2a,b,c,d

7. – Let $s_n = 2^{\lfloor n/2 \rfloor}$, so that, for every integer k, $s_{2k} = s_{2k+1} = 2^k$. Evaluate

$$\liminf s_n^{1/n}, \qquad \limsup s_n^{1/n}, \qquad \liminf \frac{s_{n+1}}{s_n}, \qquad \limsup \frac{s_{n+1}}{s_n},$$

and compare with Theorem 12.2.

8. – Suppose (s_n) and (t_n) are bounded sequences, but not necessarily non-negative and not necessarily convergent, and suppose $\limsup s_n = s$ and $\limsup t_n = t$. Is it a correct theorem that $\limsup(s_n + t_n) = s + t$? Is it a correct theorem that $\limsup(s_n t_n) = st$? This problem requires either a proof similar to that on the last homework, or a counterexample or both.

---

9. – Suppose (s_n) is a sequence with the property that for every k,

$$|s_{2k+1} - s_{2k}| < \frac{1}{k} \qquad \text{and} \qquad |s_{2k+2} - s_{2k+1}| < \frac{1}{2^k}.$$

Must it be true that (s_n) is a Cauchy sequence? (Proof, or counterexample.)

10. – Using Theorem 12.2, or any correct method, compute

$$\lim \left(\frac{(2n)!}{(n!)^2} \right)^{1/n}, \qquad \lim \left(\frac{(5n)!}{(2n)^2} \right)^{1/n}, \qquad \lim \left(\frac{(mn)!}{(n!)^m} \right)^{1/n} \quad \text{for fixed } m.$$

Hint: $(2(n+1))! = (2n+2)(2n+1)(2n)!$, etc.