Math 347 Homework 6 Due Mon., Oct. 15, 2001

1. – 14.3 – a,b,c,d (ungraded)

2. – 14.4 – a,b,c (hint for c: the root test; we've already done it in class.)

3. – 14.6

4. – 14.7 (ungraded)

5. – 14.8

6. – 14.12

7. – 16.4 - c,f

8. – (The correction.) Suppose (s_n) and (t_n) are bounded sequences, but not necessarily non-negative and not necessarily convergent, and suppose $\limsup s_n = s$ and $\limsup t_n = t$. Is it a correct theorem that $\limsup(s_n + t_n) \leq s + t$? Is it a correct theorem that $\limsup(s_n t_n) \leq st$? This problem requires either a proof similar to that on the last homework, or a counterexample or both.

9. – Observe that for every $n \in N$, there exists $r \geq 0$ so that $2^r \leq n \leq 2^{r+1} - 1$. (In fact, $r = \lfloor \log_2 n \rfloor$.) Define a sequence (s_n) as follows: if $2^r \leq n \leq 2^{r+1} - 1$ and r is even, then $s_n = 1$; if r is odd, then $s_n = 0$. Thus, for $n = 1, 2, 3, 4, 5, 6, 7$, we have $r = 0, 1, 1, 2, 2, 2, 2$ and $s_n = 1, 0, 0, 1, 1, 1, 1$. As before, let $\sigma_n = \frac{1}{n}(s_1 + \cdots + s_n)$. Compute σ_n when $n = 2^r + j$, $0 \leq j \leq 2^r - 1$ and determine $\liminf \sigma_n$ and $\limsup \sigma_n$. (Hint: first compute $\sigma_{2^{2k}}$ and $\sigma_{2^{2k+1}}$.)

10. – Construct a sequence (s_n) so that

$$\liminf \frac{s_{n+1}}{s_n} < \liminf s_n^{1/n} < \limsup s_n^{1/n} < \limsup \frac{s_{n+1}}{s_n}$$

Suggestion: Emulate the example of Homework 5, #7, but make different rules for $\frac{s_{n+1}}{s_n}$ depending on whether $2^{2k} \leq n < 2^{2k+1}$ or $2^{2k+1} \leq n \leq 2^{2k+2}$.