Math 347 **Homework 8** **Due Wed., Oct. 31, 2001**

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1. – 17.10 abc.

2. – 19.4.

3. – 19.9 (ungraded).

4. – 19.10.

5. – 20.4 and 20.8.

6. – 20.11 (ungraded).

7. – 20.12.

8. – Suppose $p(x)$ and $q(x)$ are polynomials and suppose $p(a) = q(a)$ for some $a \in \mathbf{R}$. Define

$$f(x) = \begin{cases} p(x), & \text{if } x \leq a, \\ q(x), & \text{if } x > a. \end{cases}$$

Prove carefully (that is, with an ϵ-δ argument), that f is continuous at $x = a$. You may assume without proof that p and q are continuous on $\mathbf{R}$.

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9. – 20.18. (You may use L'Hopital's Rule *informally*, to determine the limit, but you have to justify your claims rigorously.)

10. Let f be the function in 19.9; that is, $f(x) = x\ sin(\frac{1}{x})$ for $x \neq 0$ and $f(0) = 0$. We know from general principles that f is uniformly continuous on $[0, 1]$. In this problem, you will prove one instance of it directly.

Find, with proof, $\delta > 0$ with the property that for $0 \leq x, y, \leq 1$,

(*) $$|x - y| < \delta \implies |f(x) - f(y)| < \frac{1}{347}.$$

You may use the Mean Value Theorem, and you do *not* have to find the "best" (that is, the largest) δ for which (*) holds. On the other hand, you *do* have to prove it!