

1. – 23.1 aceg (ungraded).
2. – 23.1 bdfh (graded!, no answers in back)
3. – 23.4. (Note that $a_{2n} = (\frac{6}{5})^{2n}$ and $a_{2n+1} = (\frac{2}{5})^{2n+1}$.)
4. – 24.5. (ungraded)
5. – 24.6.
6. – 24.8.
7. – 24.10a.
8. – 25.6.

9. – 23.4 (continued). **Assume** the following theorem (proved below): if $\sum_{n=0}^{\infty} a_n$ is absolutely convergent, then $\sum a_{2k}$ and $\sum a_{2k+1}$ are both also absolutely convergent and

$$(*) \quad \sum_{n=0}^{\infty} a_n = \sum_{k=0}^{\infty} a_{2k} + \sum_{k=0}^{\infty} a_{2k+1}.$$

Using (*), find a closed form for the power series in 23.4c on its interval of absolute convergence.

10. – 24.14.

Proof of (*) – just for your information. Let $s_N = \sum_{n=0}^N a_n$, $S_N = \sum_{n=0}^N |a_n|$. Observe that $\sum_{n=0}^N |a_{2n}| = |a_0| + |a_2| + \cdots + |a_{2N}| \leq \sum_{n=0}^{2N} |a_n| = S_{2N}$ and since $\sum a_n$ is absolutely convergent, there exists M so that $S_n < M$ for all n . Thus the partial sums of the absolute values of $\sum a_{2n}$ are uniformly bounded and so the series is absolutely convergent. The same thing is true for $\sum a_{2n+1}$. Since absolutely convergent series are convergent, it follows that $\sum a_{2n} = L$ and $\sum a_{2n+1} = L'$ for real L, L' . Given $\epsilon > 0$, there exist N_1 and N_2 so that

$$n > N_1 \implies \left| \sum_{k=0}^n a_{2k} - L \right| < \frac{\epsilon}{2}, \quad n > N_2 \implies \left| \sum_{k=0}^n a_{2k+1} - L' \right| < \frac{\epsilon}{2},$$

Suppose $N > \max\{2N_1, 2N_2 + 1\}$. Break up the partial sums of the original sequence into the terms with even index and odd index. By the usual inequalities, we find that

$$n > N \implies \left| \sum_{k=0}^n a_k - (L + L') \right| < \epsilon.$$

QED