

Same description as before.

1. §2.5 – 2.
2. §2.5 – 6.
3. §2.5 – 8.
4. §2.5 – 14.
5. (E) Let C denote the circle $|z - i| = 1$, taken in the usual counterclockwise orientation. Compute by any correct method

$$\int_C \frac{z}{(z^2 + 1)^2} dz.$$

6. (E) Find the Taylor series for $f(z) = \frac{1+2z}{(1-z)^2}$ at $z = 0$. (Hint: either of the following identities might be useful: $1 + 2z = 1 + 2 \cdot z$; $1 + 2z = 3 + 2(z - 1)$.)
7. (E) Classify the singularity of

$$f(z) = \frac{\sin(z^3) - z^3}{z^{16}}$$

at $z = 0$ as one of {removable singularity, essential singularity, pole of order m for specific m }, and compute the residue of f at $z = 0$.

8. (E) Suppose f and g are entire functions and neither is identically zero. Suppose further that, for all z , $|f(z)| \leq |g(z)|$.
 - a. Show that the only singularities of $h = \frac{f}{g}$ are removable ones at the zeros of g . (Hint: you know something about h in a neighborhood of a zero of g .)
 - b. Prove that there is a constant c , $|c| \leq 1$ so that $f(z) = cg(z)$ for all z . (Hint: use (a) and an important theorem, applied to an entire function that is usually equal to h .)

9. (E) Suppose $f(x + iy) = u(x, y) + iv(x, y)$ is entire, and $|348u(x, y) + 2001v(x, y)| < 1$ for all (x, y) . Prove that f is constant.

10. Let C denote a contour consisting of a line segment from $1 - 2i$ to $4i$ followed by a line segment from $4i$ to $-2 - i$. Define a branch of the logarithm which is analytic on a domain containing C and use it to evaluate

$$\int_C \frac{dz}{z}$$

This problem requires both a number and a function.