Math 230 **Honors Question 2** **Due Fri., Sept. 9, 2005**

Try to follow the instructions, step by step. You will have a chance to ask me questions on Wednesday, before this is due!

a. Suppose f is a differentiable function on the interval $[0,1]$. Prove that

$$\int_0^1 f(x)\ dx = \frac{f(0)+f(1)}{2} - \int_0^1 f'(x)(x-\tfrac{1}{2})\ dx.$$

Hint: Use parts, with $u = f(x)$ and $v = x - \frac{1}{2}$, and evaluate uv carefully at the endpoints $x = 0, 1$.

b. Suppose f is a twice-differentiable function on the interval $[0,1]$. Prove that

$$\int_0^1 f(x)\ dx = \frac{f(0)+f(1)}{2} + \int_0^1 f''(x)\left(\frac{x^2-x}{2}\right)\ dx.$$

Hints: Use parts again as well as the result in (a). You should be able to guess u and v from the given formula. Some things you have to evaluate will turn out to equal zero.

c. Let $F(x) = \frac{x^2-x}{2}$. Determine the maximum and minimum of $F(x)$ on the interval $[0,1]$. Hint: if you're rusty on this, look at section 3.5.

You were supposed to have learned in the first semester that if $g(x)$ is a function that can be integrated, and if $m \le g(x) \le M$ for all x in the interval $[1,b]$, then

$$m(b-a) \le \int_a^b g(x)\ dx \le M(b-a).$$

You can find this in section 5.5 of the book. I'm *not* asking you to prove it.

d. Suppose f is a twice-differentiable function on the interval $[0,1]$ and suppose that r and s are numbers so that $|f''(x)| \le M$ for all x in $[0,1]$. Use the results of b and c to show that

$$\left|\int_0^1 f(x)\ dx - \frac{f(0)+f(1)}{2}\right| \le \frac{M}{8}.$$

e. Let k be an integer and let $f(x) = \ln(k+x)$, compute $f'(x)$ and $f''(x)$ and determine M for $f''(x)$ on $[0,1]$. Write the formula from d. as an inequality involving

$$\left|\int_0^1 \ln(x+k)\ dx - \frac{\ln(k)+\ln(k+1)}{2}\right|$$

f. (extra credit). Observe that

$$\int_0^1 \ln(x+k)\ dx = \int_k^{k+1} \ln(x)\ dx,$$

combine the inequalities from e. and exponentiate to give an estimate for $n!$ that involves

$$\frac{\left(n+\frac{1}{2}\right)^{n+\frac{1}{2}}}{e^{n-1}}$$

and various messy constants that we don't (yet) know how to write nicely.