Math 453 **Homework 1** **Due Wed., Sept. 5, 2007**

The "ungraded" problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, well, not graded. It is not necessary to submit these in your assignment. On the other hand, they are occasionally the basis for exam questions. You are always invited to work other problems as well. It may happen that part of a question is answered in the back of the book. You will not receive full credit unless you add some explanation. The symbol ($\mathcal{E}$) means that at least part of this problem appeared on an old exam, up to possible numerical alterations. The book numbers all problems in a chapter sequentially. The problems taken from the book on this assignment are all from Chapter 1. It is important to write your proofs carefully and clearly.

(ungraded) Strayer – Problems 3a, 21c, 32e, 36ab, 54ac, 61c, 64

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1. Strayer – Problem 10. (Note that there is a hint to (a) in the back of the book.)

2. Strayer – Problem 41.

3. Strayer – Problem 70. (Note that there is a hint to (b) in the back of the book.)

4. ($\mathcal{E}$) Compute $g = \gcd(77, 91)$ and $M = \mathrm{lcm}(77, 91)$ by any correct method, and find two integers m and n so that $g = 77m + 91n$.

5. ($\mathcal{E}$) True or false: if a and b are integers, and $\gcd(a, b) > 1$, then $\gcd(a, b + 1) = 1$. (I want either a short proof or a numerical counterexample.)

6. ($\mathcal{E}$) Suppose $\gcd(a, b) = 4$ and $\gcd(a, c) = 6$. Let $\gcd(b, c) = g$. What can you say about $\nu_2(g)$? What can you say about $\nu_3(g)$? What can you say about $\nu_5(g)$? (Recall that $\nu_p(n)$ is defined to be the largest power of a prime p which divides the integer n.)

7. ($\mathcal{E}$) Determine by any correct method

$$\nu_2(453!), \qquad \nu_3(453!), \qquad \nu_3\left(\binom{453}{210}\right)$$