Math 453 **Homework 2** **Due Fri., Sept. 14, 2007**

The "ungraded" problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, well, not graded. It is not necessary to submit these in your assignment. On the other hand, they are occasionally the basis for exam questions. You are always invited to work other problems as well. It may happen that part of a question is answered in the back of the book. You will not receive full credit unless you add some explanation. The symbol ($\mathcal{E}$) means that at least part of this problem appeared on an old exam, up to possible numerical alterations. The book numbers all problems in a chapter sequentially. All problems in this homework are from Chapter Two. It is important to write your proofs carefully and clearly.

(ungraded) Strayer – Problems 4ace, 28a, 29ac, 33ac.

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1. Strayer – Problem 11. (I'd organize the proof by showing that the common divisors of a and m are the same set of integers as the common divisors of b and m.)

2. Strayer – Problem 26.

3. Strayer – Problem 28b, 29b, 33b

4. ($\mathcal{E}$) The three parts here have equal weight.
   a. Compute $g = \gcd(56, 91)$.
   b. Find two specific integers m and n so that $g = 56m + 91n$.
   c. Find *all* integers (x, y) so that $g = 56x + 91y$.

5. ($\mathcal{E}$) True or false (short proofs or short refutations)
   (a) If p is a prime and n is an integer and $p^5 \mid n^2$, then $p^6 \mid n^2$.
   (b) $\nu_5(\binom{26}{13}) = 2$.

6. ($\mathcal{E}$) Find a single pair of positive integers m and n with the property that $2 * \gcd(m, n)$ is a square and $3 * \operatorname{lcm}(m, n)$ is a cube. You are not asked to find all pairs of integers. There is at least one example in which the only primes which divide m or n are 2 and 3.

7. ($\mathcal{E}$) Find by any correct method all integers x with the property that

$$x \equiv 5 \mod 18,$$

$$x \equiv 13 \mod 20.$$