Math 448 **Homework 8** **Due Friday, April 8, 2005**

In doing the integrals, you **must** justify your steps in explaining why part of the contour integral might go to zero as the contour as whole changes. On the other hand, it's acceptable to quote results from the book or class, if you do so specifically. Please note that three of the problems count double.

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1. (ungraded) §2.5 – 13.
2. (ungraded) §2.6 – 3.
3. (ungraded) §2.6 – 9.

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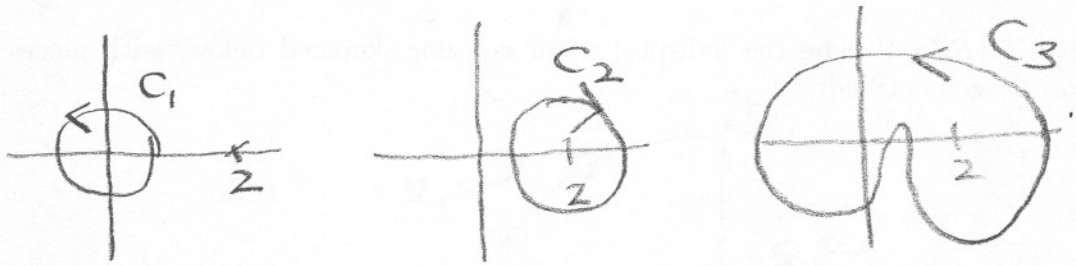
4. Compute

$$\int_{C_1} \frac{1}{z^4(z-2)}\,dz, \qquad \int_{C_2} \frac{1}{z^4(z-2)}\,dz, \qquad \int_{C_3} \frac{1}{z^4(z-2)}\,dz;$$

where C_1, C_2 and C_3 are sketched below.



5.,6. (counts as two problems) ($\mathcal{E}$) Compute, with explanation,

$$\int_0^\infty \frac{x^2}{(x^2+9)^3}\,dx.$$

7.,8. (counts as two problems) ($\mathcal{E}$) Evaluate

$$\int_0^{2\pi} \frac{\cos\theta}{5+3\cos\theta}\,d\theta.$$

9.,10. (counts as two problems) ($\mathcal{E}$) Evaluate

$$\int_0^\infty \frac{x\sin x}{x^2+1}\,dx$$

by integrating an appropriate function over an appropriate contour.

11. ($\mathcal{E}$) Let C be any simple closed curve in the complex plane, taken in the positive orientation. We define a function $f(z)$ for all $z \notin C$ as follows:

$$f(z) = \frac{1}{2\pi i} \int_C \frac{e^{2\zeta}}{(\zeta - z)^3}\, d\zeta.$$

Compute $f(z)$. (Your answer will depend on whether z is inside C or outside C.)

12a. Suppose k and ℓ are non-negative integers and $a \in \mathbf{C}$ and let

$$f(z) = \frac{1}{(z+a)^\ell}.$$

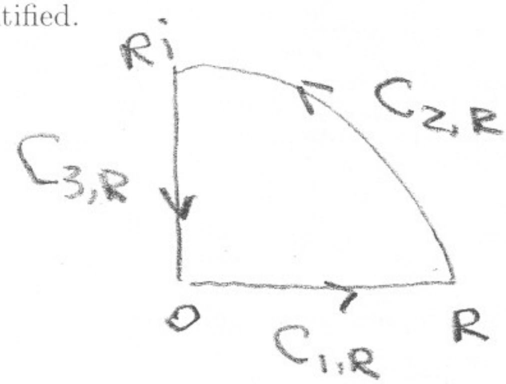
Prove by induction (or otherwise) that the k-th derivative of f is equal to

$$(-1)^k \frac{(\ell + k - 1)!}{(\ell - 1)!} \cdot \frac{1}{(z+a)^{\ell+k}}.$$

12b. Suppose n is a positive integer. Compute

$$\int_0^\infty \frac{dx}{(x^2+1)^n}.$$

13. Let C_R ($R > \sqrt{2}$) be the half-protractor contour sketched below, with pieces $C_{1,R}$, $C_{2,R}$ and $C_{3,R}$ identified.



13a. Show that

$$f(z) = \frac{z^2}{z^4+4}$$

has exactly one singularity within C_R and compute its residue there.

13b. Find a complex number λ so that

$$\int_{C_{3,R}} f(z)\, dz = \lambda \int_{C_{1,R}} f(z)\, dz$$

(This will involve carefully parameterizing both line segments.)

13c. Put everything together to evaluate

$$\int_0^\infty \frac{x^2}{1+x^4}\, dx.$$