Math 423 - Homework 11 Tip Sheet - HW due 12/6/04.

1. Unbelievably enough, straightforward!

2a) The easiest way to express the joint trigonometric inverse is as follows: if $a^2+b^2=1$ and $a=\cos\theta$, $b=\sin\theta$, you can write $a+ib=e^{i\theta}$, and $\theta=\arg(a+ib)$. (This is, of course, only defined up to a multiple of 2π.) This problem turned out to be uglier than I thought

b) It's also not correct, replace "isometric" by "locally isometric"

3.a) Computer algebra may help

~~5~~ 4 b) For all the curvature computations, t is a constant - it's really a parameter. The point of (d) is that the unit normal

$$\vec{\mathcal{U}}_t=\frac{(\vec{X}_t)_u\times(\vec{X}_t)_v}{\|(\vec{X}_t)_u\times(\vec{X}_t)_v\|}$$

doesn't actually depend on t.

c). As a computational aid, you should find that there exist vectors $\vec{W}(u,v)$ and $\vec{Z}(u,v)$ so that

$$(\vec{X}_t)_u=\cos t\,\vec{W}+\sin t\,\vec{Z}$$
$$(\vec{X}_t)_v=\sin t\,\vec{Z}-\cos t\,\vec{W}$$

It's helpful to calculate $\vec{W}\cdot\vec{W}$, $\vec{W}\cdot\vec{Z}$ and $\vec{Z}\cdot\vec{Z}$ first

d) (b) is much harder for "isometry" than "local isometry", and I'll be happy if you leave it at that.

6 + 7 Both are ... challenging. If you want to do some library work, #6 (5.6, #19) is known as Catalan's Theorem (1842)

#7 is just ugly. I'll provide a complete solution in the special case $(g(u),h(u))=(F(u),u)$.

For grad credit, do either 6 or 7
(bonus)