Math 210 Instructor: Ryan Hubscher
Theory of Interest Spring, 2008

Exam # 1 Version a
Tuesday February 26, 2008

1. Given $i^{(4)} = 10\%$, evaluate $i, i^{(12)}, \delta$, and $d^{(2)}$.

$$i = (1.025)^4 - 1 = \mathbf{10.3813\%}$$
$$i^{(12)} = 12\left[(1.025)^{(1/3)} - 1\right] = \mathbf{9.9178\%}$$
$$\delta = \ln((1.025)^4) = \mathbf{9.8770\%}$$
$$d^{(2)} = 2\left[1 - (1.025)^{-2}\right] = \mathbf{9.6371\%}$$

2. Given $i^{(4)} = 10\%$, evaluate $a_{\overline{5}|}, a^{(12)}_{\overline{5}|}, \bar{s}_{\overline{5}|}$, and $\ddot{s}^{(2)}_{\overline{5}|}$.

$$a_{\overline{5}|} = \frac{1 - (1.025)^{-20}}{.103813} = \mathbf{3.7541}$$
$$a^{(12)}_{\overline{5}|} = \frac{1 - (1.025)^{-20}}{.099178} = \mathbf{3.9296}$$
$$\bar{s}_{\overline{5}|} = \frac{(1.025)^{20} - 1}{.098770} = \mathbf{6.4657}$$
$$\ddot{s}^{(2)}_{\overline{5}|} = \frac{(1.025)^{20} - 1}{.096371} = \mathbf{6.6266}$$

3. You deposit \$100 into account A and \$80 into account B. Account A earns an effective annual rate of discount $d = 10\%$. Account B earns a nominal annual rate of interest compounded quarterly of $i^{(4)}$. Five years later, the two accounts have the same accumulated value. Find $i^{(4)}$.

$$100(1 - .1)^{-5} = 80\left(1 + \frac{i^{(4)}}{4}\right)^{5\cdot 4}$$
$$i^{(4)} = 4\left[\left(\frac{100}{80}(0.9)^{-5}\right)^{1/20} - 1\right]$$
$$= \mathbf{15.2837\%}$$

4. You borrow \$1000 today. You will repay your loan over 20 years with a payment of x at the end of each year for the first 10 years and $3x$ at the end of each year for the next 10 years. Find x if $i = 7\%$.

0	1	2	$\cdots$	10	11	$\cdots$	20
(1000)	$3x$	$3x$	$\cdots$	$3x$	$3x$	$\cdots$	$3x$
	$(2x)$	$(2x)$	$\cdots$	$(2x)$			

$$1000 = 3x\, a_{\overline{20}|} - 2x\, a_{\overline{10}|}$$
$$1000 = 31.7820x - 14.0472x$$
$$x = \mathbf{56.39}$$

5. You lend a friend \$15,000, and you charge her interest at an effective annual rate of $i = 6\%$. She will repay the loan with payments of \$1000 paid at the end of each year until the loan is repaid in full. Find the number of regular loan payments.

$$15000 = 1000\, a_{\overline{n}|\,0.06}$$

$$n = -\frac{\ln(1 - (15)(.06))}{\ln(1.06)} = 39.52$$

Thus, there will be **39** regular payments.

6. The price of a stock is \$40 per share. The stock pays a dividend at the end of each year, and the first dividend, x, will be paid one year from now. Each year, the dividend is expected to increase at a rate of $r = 7\%$ per year. If The yield rate on the stock is $i = 12\%$. Evaluate x using the dividend discount model.

$$40 = \frac{x}{.12 - .07}$$

$$x = \mathbf{2}$$

7. An investment account earns 3% the first year, 10% the second year, and $X\%$ the third year. Find X if the three year average interest rate is 8%.

$$(1.03)(1.10)(1 + X) = (1.08)^3$$

$$X = \mathbf{11.1838}\%$$

8. Suppose that the accumulation function for an account is $a(t) = (1 + it^2)$. At time 0, you invest \$100 in this account. If the value in the account at time 10 is \$400, what is i?

$$A(10) = A(0)\, a(10)$$

$$400 = 100(1 + i(10^2))$$

$$i = \mathbf{0.03}$$

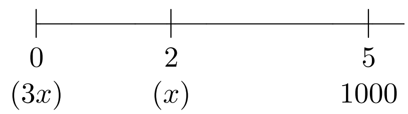
9. You deposit \$1000 today, withdraw x two years from now, deposit \$5000 three years from now, and there is x left in the account six years from now. If the account earns a nominal annual rate of interest compounded semiannually of 10%, find the value of x.

Time (Half Years)	0	4	6	12
Payment	1000	(x)	5000	(x)

$$1000(1.05)^4 + 5000(1.05)^{-2} = x + x(1.05)^{-8}$$

$$x = \mathbf{3429.46}$$

10. You need \$1000 at time 5. To save for this amount, you invest $3x$ at time 0 and x at time 2. The force of interest is $\delta_t = 0.01t^2$. Find x.



$$3x\,e^{\int_0^5 0.01t^2\,dt} + x\,e^{\int_2^5 0.01t^2\,dt} = 1000$$

$$3x\,e^{0.41667} + x\,e^{0.39} = 1000$$

$$x = \mathbf{165.90}$$