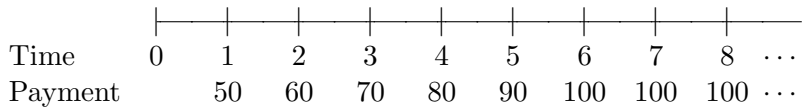
Math 210 Instructor: Ryan Hubscher
Theory of Interest Spring, 2008

Exam # 2 Version a
Thursday April 3, 2008

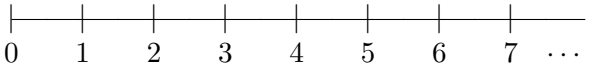
1. Find the present value of a perpetuity-immediate that makes the following payments:



Time	0	1	2	3	4	5	6	7	8	$\cdots$
Payment		50	60	70	80	90	100	100	100	$\cdots$

After time 6, this perpetuity continues to pay 100 forever. The effective interest rate is $i = 5\%$.

This cash flow can be expressed



Time	0	1	2	3	4	5	6	7	$\cdots$
Level Perpetuity		100	100	100	100	100	100	100	$\cdots$
Decreasing Annuity		(50)	(40)	(30)	(20)	(10)			

$$100a_{\overline{\infty}|} - 10(Da)_{\overline{5}|}$$
$$= \frac{100}{.05} - 10\left(\frac{5 - a_{\overline{5}|}}{.05}\right) = \mathbf{1865.90}$$

2. A 10-year annuity makes payments continuously at a rate $10 + t$. The force of interest varies continuously at a rate of $\frac{1}{10+t}$ where t measures time in years. Find the accumulated value of this annuity at time 10.

$$\int_t^{10} \frac{1}{10+s}\, ds = \ln(10+s)\Big|_t^{10} = \ln\left(\frac{20}{10+t}\right)$$

$$\int_0^{10} (10+t)e^{\int_t^{10} \frac{1}{10+s}\, ds}\, dt = \int_0^{10} (10+t)\left(\frac{20}{10+t}\right)\, dt = \int_0^{10} 20\, dt = \mathbf{200}$$

3. A 30-year loan involves monthly amortization payments of \$500 at the end of each month. The nominal annual interest rate is 7.2% convertible monthly. Find the initial amount of the loan (i.e., the balance at time 0)

$$L = OB_0 = 500a_{\overline{360}|.006} = \mathbf{73660.68}$$

4. For problem 3, find the amount of principal repaid with the 100^{th} payment

$$OB_{99} - OB_{100} = 500a_{\overline{261}|} - 500a_{\overline{260}|} = 500v_{.006}^{261} = \mathbf{104.93}$$

5. A 20-year loan of \$100,000 has a nominal annual rate of interest of 8.4% convertible monthly. The lender will receive interest payments only at the end of each month for 20 years and a lump-sum principal repayment at the end of year twenty in addition to the interest payment at that time. The borrower plans to accumulate the lump-sum

payment by means of 240 level payments into a sinking fund at the end of each month earning an nominal annual rate of interest of 6% convertible monthly. What is the net outstanding balance on this loan at the end of the 6^{th} month?

$$100000\left(1-\frac{s_{\overline{6}|.005}}{s_{\overline{240}|.005}}\right)=\mathbf{98685.07}$$

6. Carl purchases an investment for 1,000. The investment pays Carl 80 at the end of every year for 20 years. At the date of the last payment, the original principal of 1,000 is returned to Carl in addition to the final payment of 80. If Carl can reinvest his returns at an annual effective rate of interest of 6%, what is the yield on Carl's investment?

   The accumulated value of Carl's investment at time 20 is $80s_{\overline{20}|0.06}+1000=3942.847$. The Yield on this investment is the growth rate from Carl's original investment of 1000 to the accumulated value 3942.847. Thus, the yield is the solution to the equation $1000(1+j)^{20}=3942.847$. which results in a yield of $j=\mathbf{7.10}\%$.

7. Find the price of a \$2,000 par value 20-year 5% bond with semiannual coupons bought to yield 9% convertible semiannually.

$$2000v_{.045}^{40}+50a_{\overline{40}|.045}=\mathbf{1,263.94}$$

   Using the BA II PLUS TVM Worksheet:
   [2ND] [CLR TVM] 40 [N] 4.5 [I/Y] 50 [+|-] [PMT] 2000 [+|-] [FV] [CPT] [PV]
   Result: **1,263.94**

8. Suppose a project requires you to invest \$100 now and \$99 two years from now. The project returns \$200 one year from now. Find all the yield rates of this project.

$$-100+\frac{200}{1+i}-\frac{99}{(1+i)^2}=0$$

$$100(1+i)^2-200(1+i)+99=0$$

$$100+200i+100i^2-200-200i+99=0$$

$$100i^2-1=0$$

$$i=\pm\mathbf{10}\%$$

9. Suppose the nominal annual rate of interest is 6% compounded semiannually. Find the NPV of the cash flow in problem 8.

$$-100+\frac{200}{(1.03)^2}-\frac{99}{(1.03)^4}=\mathbf{.55896}$$

10. On 1/1/08, you deposit \$3,000 into an account. At the end of each of the next four calendar quarters, the value of the account and the deposit/withdrawal activity is as follows:

Date	Account Value	Activity
3/31/08	\$3,500	\$200 withdrawal
6/30/08	3,100	100 deposit
9/30/08	3,400	4,000 deposit
12/31/08	8,000	–

(The account values represent the amount in the account immediately before the deposit or withdrawal activity on that date.) Find the time-weighted rate of return on the account during 2008.

$$\left(\frac{35}{30}\right)\left(\frac{31}{35-2}\right)\left(\frac{34}{31+1}\right)\left(\frac{80}{34+40}\right)-1=\mathbf{25.89}\%$$

11. You invest \$5,000 in a fund on 1/1/07 (month/day/year). On 5/1/07, you deposit another \$3,000 into the fund. On 12/31/07, your fund is worth \$9,000. What was the annual dollar-weighted rate of return on your investment?

$$\frac{9000-5000-3000}{5000+\frac{2}{3}3000}=\mathbf{14.286}\%$$