

# Examples 8

Math 210

April 18, 2008

## 1 Read Your Book

One of your classmates asked for more examples. As indicated by the web site, here is where you can find them.

<i>Topic</i>	<i>Broverman</i>	<i>Number of Pages</i>
Callable bonds	4.3.1	5
Investment year method and Portfolio year methods	5.3.1	2.5
Spot rates	6.1 (optional 6.2)	7
Forward rates	6.3	3
<i>Total</i>		17.5

You can just skim through 6.1 and focus mainly on the examples. Understanding table 6.3 will help you finish problem 7.

## 2 Forward Rates

I admit, forward rates may be the most challenging part of this reading. However, the concept is simple. Here is a quick explanation.

Suppose you are given the following term structure of interest rates.

<i>Length of Investment</i>	<i>Spot Rate</i>
1	0.03
2	0.04
3	0.06
4	0.07

In the real world, these rates come from data concerning market trades. We look at how much people pay for various strips, then we use their purchase price to determine the yield rate for that zero coupon bond. This yield rate is called a spot rate. Hidden within these spot rates, we can discover what investors in the market believe spot rates will be next year. These are called forward rates. The forward rates one year from now are given by  $i_{1,2}, i_{1,3}, i_{1,4}$ . These rates are what we think the one, two, and three year spot rates will be

after one year has passed. To compute them, solve these equations.

$$\begin{aligned}(1.03)(1 + i_{1,2})^1 &= (1.04)^2 \\ (1.03)(1 + i_{1,3})^2 &= (1.06)^3 \\ (1.03)(1 + i_{1,4})^3 &= (1.07)^4\end{aligned}\tag{1}$$

Both sides of equation (1) tell us how much a dollar is hoped to accumulate in two years. On the left side, the accumulation factor from time 0 to 1 is 1.03 and the accumulation factor from time 1 to 2 is  $(1 + i_{1,2})$ . On the right side, the accumulation factor from time 0 to 2 is  $(1.04)^2$ .

To compute the forward rates two years from now, solve these equations.

$$\begin{aligned}(1.04)^2(1 + i_{2,3})^1 &= (1.06)^3 \\ (1.04)^2(1 + i_{2,4})^2 &= (1.07)^4\end{aligned}$$

To compute the one year forward rate three years from now, solve

$$(1.06)^3(1 + i_{3,4})^1 = (1.07)^4$$

To review, the  $k$  year spot rate that investors believe will be in force  $n$  years from now is called the  $k$ -year forward rate of interest  $n$  years from now. Broverman uses the confusing term  $n$ -year forward,  $k$ -year interest rate. The symbol for this rate is  $i_{n,n+k}$  because it is the expected rate from time  $n$  to time  $n + k$ .

In general, if  $s_n$  is the current  $n$ -year spot rate, the  $k$ -year forward rate  $n$  years from now is calculated by examining

$$(1 + s_n)^n(1 + i_{n,n+k})^k = (1 + s_{n+k})^{n+k}\tag{2}$$

Solving for  $i_{n,n+k}$  gives us

$$i_{n,n+k} = \left[ \frac{(1 + s_{n+k})^{n+k}}{(1 + s_n)^n} \right]^{1/k} - 1\tag{3}$$

You shouldn't try to memorize equation (3). Rather, know that it comes from (2).