

Homework Assignment # 3 (max. points = 20)
Due at the beginning of class on Thursday February 7, 2008

Please show your work - enough to show that you understand how to do the problem. Circle your final answer. Full credit can only be given only if the answer and work leading to the answer are correct.

1. Find the accumulated value of an annuity that pays \$100 at the end of each year for 20 years. The effective annual rate of interest is 8%.

$$100 s_{\overline{20}|} = 100 \frac{(1.08)^{20} - 1}{0.08} = \mathbf{4576.20}$$

2. You want to accumulate \$1000 at the end of 15 years. How much must you invest at the end of each year if the effective annual rate of interest is 7%?

$$1000 = x s_{\overline{15}|.07}$$

$$x = \mathbf{39.79}$$

3. If a person invests \$1000 today at 8% per annum convertible quarterly, how much can be withdrawn at the end of every quarter to use up the fund exactly at the end of 8 years?

$$1000 = x a_{\overline{32}|.02}$$

$$x = \mathbf{42.61}$$

4. Over the next 20 years, you deposit money into a retirement account at the end of each year according to the following schedule:

Time	Amount Invested Each Year
1-5	\$2000
6-10	3000
11-20	5000

The effective annual rate of interest is 9%. Find the accumulated value of your account at time 20.

$$\begin{aligned} & 2000 s_{\overline{20}|} + 1000 s_{\overline{15}|} + 2000 s_{\overline{10}|} \\ & = 102,320.24 + 29,360.92 + 30,385.86 = \mathbf{162,067.01} \end{aligned}$$

5. You borrow \$5000 today. You will repay your loan over 20 years with a payment of $2X$ at the end of each year for the first 10 years and X at the end of each year for the next 10 years. Find X if $i = 10\%$.

$$5000 = X a_{\overline{10}|} + X a_{\overline{20}|}$$

$$5000 = 6.1446X + 8.5136X$$

$$X = \mathbf{341.1076}$$

6. An annuity pays \$500 every six months for 5 years. The annual rate of interest is 8% convertible semiannually. Find each of the following:

- A. The PV of the annuity six months (one period) before the first payment,
 B. The PV of the annuity on the day of the first payment,
 C. The FV of the annuity on the day of the last payment,
 D. And the FV of the annuity six months after the last payment.

$$\text{A. } 500 a_{\overline{10}|.04} = \mathbf{4,055.45}$$

$$\text{B. } 500 \ddot{a}_{\overline{10}|.04} = \mathbf{4,217.67}$$

$$\text{C. } 500 s_{\overline{10}|.04} = \mathbf{6,003.05}$$

$$\text{D. } 500 \ddot{s}_{\overline{10}|.04} = \mathbf{6,243.18}$$

7. You can receive one of the following two sets of cash flows. Under Option A, you will receive \$5,000 at the end of each of the next 10 years. Under Option B, you will receive X at the beginning of each year, forever. The annual effective rate of interest is 10%. Find the value of X such that you are indifferent between these two options.

$$PV_A = PV_B$$

$$5000 a_{\overline{10}|} = X \left(\frac{1}{d} \right) = X \left(\frac{1}{i} + 1 \right)$$

$$30,722.84 = 11X$$

$$X = \mathbf{2,792.99}$$

8. A special perpetuity pays \$200 at the end of each year for the first 10 years and \$100 at the end of each year thereafter. If the effective annual rate of interest is 10%, find the present value of the perpetuity.

$$100 a_{\overline{\infty}|} + 100 a_{\overline{10}|}$$

$$= 100 \left(\frac{1}{.1} \right) + 100 \left(\frac{1 - v^{10}}{.1} \right)$$

$$= \mathbf{1614.46}$$

9. You will retire in 30 years. At the beginning of each month until you retire, you will invest X earning interest at 9% convertible monthly. Starting at year 30, you will withdraw \$4000 at the beginning of each month for the next 15 years. Also, starting at year 30, your fund will only earn interest at 6% convertible monthly. Find X such that your account will be empty after the last withdrawal.

$$X \ddot{s}_{\overline{360}|.0075} = 4000 \ddot{a}_{\overline{180}|.005}$$

$$X \left(\frac{1.0075^{360} - 1}{.0075/1.0075} \right) = 4000 \left(\frac{1 - 1.005^{-180}}{.005/1.005} \right)$$

$$X = \mathbf{258.276}$$

10. Annuities X and Y provide the following payments:

End of Year	Annuity X	Annuity Y
1-10	1	K
11-20	2	0
20-30	1	K

Annuities X and Y have equal present values and an annual effective interest rate of i such that $v^{10} = 1/2$. Determine K .

$$PV_X = PV_Y$$

$$a_{\overline{30}|} + a_{\overline{10}|} v^{10} = K (a_{\overline{30}|} - a_{\overline{10}|} v^{10})$$

$$\frac{1 - v^{30}}{i} + \frac{1 - v^{10}}{i} v^{10} = K \left(\frac{1 - v^{30}}{i} - \frac{1 - v^{10}}{i} v^{10} \right)$$

$$1 - v^{30} + v^{10} - v^{20} = K (1 - v^{30} - v^{10} + v^{20})$$

$$1 - \frac{1}{8} + \frac{1}{2} - \frac{1}{4} = K \left(1 - \frac{1}{8} - \frac{1}{2} + \frac{1}{4} \right)$$

$$K = 1.8$$