Math 210 Instructor: Ryan Hubscher
Theory of Interest Spring, 2008

Homework Assignment # 7 (max. points = 20)
Due at the beginning of class on Thursday March 27, 2008

Please show your work - enough to show that you understand how to do the problem. Circle your final answer. Full credit can only be given only if the answer and work leading to the answer are correct.

1. A project requires the following contributions and results in the following returns. Find the NPV of this project if the effective annual rate of interest is $i = 6\%$. Also, find the IRR of this project.

Year	Contributions	Returns
0	2,000	0
1	1,500	0
2	500	1,500
3	500	1,500
4	500	1,500
5	0	1,500

BA II PLUS CF, NPV, and IRR Worksheets To solve this problem with the BA II PLUS, Enter the net cashflow using the CF worksheet. Then compute the NPV and IRR using the NPV and IRR worksheets.

Keystrokes	*Display*	
CF 2ND [CLR WORK]	CFo=	0
2000 +|- ENTER	CFo=	-2,000
↓ 1500 +|- ENTER	C01=	-1,500
↓ ↓ 1000 ENTER	C02=	1,000
↓ ↓ 1000 ENTER	C03=	1,000
↓ ↓ 1000 ENTER	C04=	1,000
↓ ↓ 1500 ENTER	C05=	1,500
NPV 6 ENTER	I=	6
↓ CPT	NPV=	**227.5023**
IRR CPT	IRR=	**8.1835**

2. Suppose a project requires you to invest \$1,000 now and \$1,045 two years from now. The project returns \$2,050 one year from now. Find all the yield rates of this project.

$$-1000 + \frac{2050}{1+i} - \frac{1045}{(1+i)^2} = 0$$

$$(1+i)^2 - 2.050(1+i) + 1.045 = 0$$

$$[(1+i) - 1.1][(1+i) - .95] = 0$$

$$i = \mathbf{10}\% \text{ or } i = -\mathbf{5}\%$$

3. For problem 2, find the range of yield rates that will result in a positive net present value.

$$NPV(i) = -1000 + \frac{2050}{1+i} - \frac{1045}{(1+i)^2} > 0 \text{ whenever } \mathbf{-.05} < i < \mathbf{.1}$$

4. You invest \$10,000 in a fund on 1/1/07 (month/day/year). On 6/1/07, you deposit another \$3,000 into the fund. On 10/1/07, you withdraw \$4,000 from the fund. On 12/31/07, your fund is worth \$14,000. What was the annual dollar-weighted rate of return on your investment?

$$\frac{14 - (10 + 3 - 4)}{10 + 3\left(\frac{7}{12}\right) - 4\left(\frac{3}{12}\right)} = \mathbf{46.51}\%$$

5. You invest \$5,000 in a fund on 1/1/07. On 3/1/07, you withdraw X from the fund. On 7/1/07, you deposit \$2,500 into the fund. On 9/1/07, you withdraw \$1,000 from the fund. On 12/31/07, your fund is worth \$5,700. The annual dollar-weighted rate of return on your investment was 12.0%. Find X.

$$\frac{5700 - (5000 - X + 2500 - 1000)}{5000 - X\left(\frac{10}{12}\right) + 2500\left(\frac{6}{12}\right) - 1000\left(\frac{4}{12}\right)} = .12$$

$$X = \mathbf{1372.73}$$

6. On 1/1/08, you deposit \$20,000 into an account. At the end of each of the next four calendar quarters, the value of the account and the deposit/withdrawal activity is as follows:

Date	Account Value	Activity
3/31/08	\$18,000	\$3,000 withdrawal
6/30/08	20,500	5,000 deposit
9/30/08	22,000	4,000 withdrawal
12/31/08	21,000	–

(The account values represent the amount in the account immediately before the deposit or withdrawal activity on that date.) Find the time-weighted rate of return on the account during 2008.

$$\left(\frac{18}{20}\right)\left(\frac{20.5}{18-3}\right)\left(\frac{22}{20.5+5}\right)\left(\frac{21}{22-4}\right) - 1 = \mathbf{23.80}\%$$

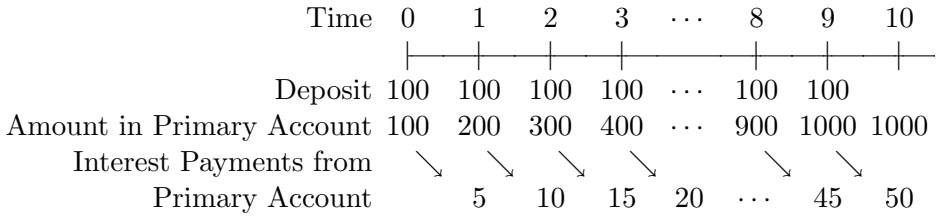
7. On 1/1/07, you deposit \$4,000 into an account. On 5/18/07, your account is worth \$5,000, and you then either deposit or withdraw (you'll have to determine which) X into/from the account. On 12/31/07, your account is worth \$5,200. Your time-weighted rate of return on the account during 2007 was 40%. Find X (and identify whether X was deposited or withdrawn).

$$\left(\frac{5000}{4000}\right)\left(\frac{5200}{5000+X}\right) = 1.4$$

$$X = -357.14$$

This indicates a **withdrawal of \$357.14**.

8. James invests 100 at the beginning of each year for 10 years at an effective annual rate of 5%. He reinvests the interest payments at an annual effective rate of 7%. Find the accumulated value of James's account one year after his last deposit.



At time 10, James will have 1000 from deposits, and the accumulated value of his interest will be the accumulated value of an increasing 10-year annuity-immediate making payments 5, 10, ..., 50 invested at 7% annual interest.

$$1000 + 5(Is)_{\overline{10}|0.07}$$

$$= 1000 + 5\left(\frac{s_{\overline{11}|0.07} - 11}{0.07}\right) = \mathbf{1341.686}$$

9. Tom purchases an investment for 5,000. The investment makes payments to Tom of 250 at the end of every year for 15 years. At the date of the last payment, the original principal of 5,000 is returned to Tom in addition to the final payment of 250. If Tom can reinvest his returns at an annual effective rate of interest of 7%, what is the yield rate on Tom's investment?

The accumulated value of Tom's investment at time 15 is $250s_{\overline{15}|0.07}+5000 = 11282.256$. The Yield rate on this investment is the growth rate from Tom's original investment of 5000 to the accumulated value 11282.256. Thus, the yield rate is the solution to the equation $5000(1+j)^{15} = 11282.256$. which results in a yield rate of $j = \mathbf{5.575}\%$.

10. Jerry lends 10,000 to Oscar at an annual rate $i^{(12)} = 6\%$. Oscar will repay this loan with level payments at the end of each month for the next 10 years. At what rate (expressed as $j^{(12)}$) must Jerry be able to reinvest the loan payments so that the yield rate will be 8% compounded monthly?

Oscar's monthly payment will be

$$\frac{10000}{a_{\overline{120}|0.005}} = 111.0205$$

These payments of 111.021 per month constitute an annuity-immediate for Jerry. As Jerry receives these payments, he will reinvest them at a monthly rate of interest x. At year 15, these payments will accumulate to

$$111.0205s_{\overline{120}|x}$$

At a yield rate of 8% compounded monthly, Jerry's investment of 10000 must grow to $10000(1 + \frac{0.08}{12})^{120} = 22196.4023$ Therefore, The monthly reinvestment rate is the solution to the equation

$$111.0205s_{\overline{120}|x} = 22196.4023$$

This equation can be solved on a financial calculator using the TVM (Time-Value of Money) functions or on Excel using the rate() function. On the TI BA II Plus, use the following keystrokes:

Keystrokes	*Display*	
2ND [CLR TVM]		
120 N	N=	120
22196.4023 FV	FV=	22,196.4023
111.0205 +|- PMT	PMT=	-111.0205
CPT I/Y	I/Y=	0.7979

So the monthly effective rate of interest is $x = 0.7979\%$. This means that the annualized yield rate is $j^{(12)} = 12x = \mathbf{9.5748}\%$.