Math 210 Instructor: Ryan Hubscher
Theory of Interest Spring, 2008

Homework Assignment # 8 (max. points = 20)
Due at the beginning of class on Thursday April 10, 2008

Please show your work - enough to show that you understand how to do the problem. Circle your final answer. Full credit can only be given only if the answer and work leading to the answer are correct.

1. Find the price of these bonds, all redeemable at par, and order them from lowest price to greatest price. These bonds make semiannual payments.

   (a) A 10-year 100, 5% bond yielding 7.2%

   $$\begin{aligned} \text{Basic Formula:} &\quad 100v_{.036}^{20} + 2.5a_{\overline{20}|.036} = \mathbf{84.51} \\ \text{Premium Discount Formula:} &\quad 100 + (2.5 - 3.6)a_{\overline{20}|.036} = \mathbf{84.51} \\ \text{Makeham's Formula:} &\quad 100v_{.036}^{20} + \tfrac{2.5}{3.6}(100 - 100v_{.036}^{20}) = \mathbf{84.51} \end{aligned}$$

   BA II PLUS TVM Worksheet To solve this problem with the BA II PLUS TVM functions, tell the calculator that $N = 20$, $I/Y = 3.6\%$, $PMT = -2.5$, and $FV = -100$. Then tell the calculator to compute the PV. To realize this, use the following keystrokes:

Keystrokes	*Display*	
[2ND] [CLR TVM]		
20 [N]	N=	20
3.6 [I/Y]	I/Y=	3.6
2.5 [+|-] [PMT]	PMT=	-2.5
100 [+|-] [FV]	FV=	-100
[CPT] [PV]	PV=	**84.51**

   BA II PLUS Bond Worksheet To solve this problem with the Bond Worksheet, tell the calculator that the settlement date is 1/01/2000, the annualized coupon is 5 (the calculator will know that this coupon is paid in two semiannual installments of 2.5 each), the redemption date is 1/01/2010, the redemption value is 100, the method of counting days is 360, the number of coupons per year is 2, and the annualized yield is 7.2. Then compute the price.

Keystrokes	*Display*	
[2ND] [BOND] [2ND] [CLR WORK]		
1.0100 [ENTER]	SDT=	1-01-2000
[↓] 5 [ENTER]	CPN=	5
[↓] 1.0110 [ENTER]	RDT=	1-01-2010
[↓] 100 [ENTER]	RV=	100
[↓] [2ND] [SET] (if necessary)	360	
[↓] [2ND] [SET] (if necessary)	2/Y	
[↓] 7.2 [ENTER]	YLD=	7.2
[↓] [CPT]	PRI=	**84.51**

(b) A 10-year 100, 5.5% bond yielding 7.7%

$$100v_{.0385}^{20} + 2.75a_{\overline{20}|.0385} = \mathbf{84.85}$$

Using the BA II PLUS TVM Worksheet:
[2ND] [CLR TVM] 20 [N] 3.85 [I/Y] 2.75 [+|-] [PMT] 100 [+|-] [FV] [CPT] [PV]
Result: **84.85**

(c) A 12-year 100, 5% bond yielding 7.2%

$$100v_{.036}^{24} + 2.5a_{\overline{24}|.036} = \mathbf{82.52}$$

Using the BA II PLUS TVM Worksheet:
[2ND] [CLR TVM] 24 [N] 3.6 [I/Y] 2.5 [+|-] [PMT] 100 [+|-] [FV] [CPT] [PV]
Result: **82.52**

(d) A 12-year 100, 5.5% bond yielding 7.7%

$$100v_{.0385}^{24} + 2.75a_{\overline{24}|.0385} = \mathbf{82.97}$$

Using the BA II PLUS TVM Worksheet:
[2ND] [CLR TVM] 24 [N] 3.85 [I/Y] 2.75 [+|-] [PMT] 100 [+|-] [FV] [CPT] [PV]
Result: **82.97**

2. For each of the bonds in problem 1, find the price-plus-accrued and quoted price of these bonds after 3.25 years if the market yield for each bond dropped to 6% compounded semiannually.

(a) A 6.75-year 100, 5% bond yielding 6%

Price at date of previous coupon	$P_0 = 100v_{.03}^{14} + 2.5a_{\overline{14}|.03} = 94.3520$
Time since previous coupon	$t = (.25 \text{ years})(2 \text{ coupon periods/year})$
	$= .5$ coupon periods
Price-plus-accrued	$P_t = P_0(1+j)^t = 94.3520(1.03)^{.5} = \mathbf{95.7568}$
Accrued Coupon	$Fr_t = t \cdot Fr = .5(2.5) = 1.25$
Quoted Price	$price_t = 95.7568 - 1.25 = \mathbf{94.5068}$

BA II PLUS Bond Worksheet:

Keystrokes	*Display*
[2ND] [BOND] [2ND] [CLR WORK]	
4.0103 [ENTER]	SDT= 4-01-2003
[↓] 5 [ENTER]	CPN= 5
[↓] 1.0110 [ENTER]	RDT= 1-01-2010
[↓] 100 [ENTER]	RV= 100
[↓] [2ND] [SET] (if necessary)	360
[↓] [2ND] [SET] (if necessary)	2/Y
[↓] 6 [ENTER]	YLD= 6
[↓] [CPT]	PRI= **94.5068**
[↓]	AI= 1.25

$$\textit{price-plus-accrued}_t = 94.5068 + 1.25 = \mathbf{95.7568}$$

(b) A 6.75-year 100, 5.5% bond yielding 6%

Price at date of previous coupon	$P_0 = 100v_{.03}^{14} + 2.75a_{\overline{14}\rceil.03} = 97.1760$
Time since previous coupon	$t = (.25 \text{ years})(2 \text{ coupon periods/year})$
	$= .5$ coupon periods
Price-plus-accrued	$P_t = P_0(1+j)^t = 97.1760(1.03)^{.5} = \mathbf{98.6229}$
Accrued Coupon	$Fr_t = t \cdot Fr = .5(2.75) = 1.375$
Quoted Price	$price_t = 98.6229 - 1.375 = \mathbf{97.2479}$

(c) A 8.75-year 100, 5% bond yielding 6%

Price at date of previous coupon	$P_0 = 100v_{.03}^{18} + 2.5a_{\overline{18}\rceil.03} = 93.1232$
Time since previous coupon	$t = (.25 \text{ years})(2 \text{ coupon periods/year})$
	$= .5$ coupon periods
Price-plus-accrued	$P_t = P_0(1+j)^t = 93.1232(1.03)^{.5} = \mathbf{94.5098}$
Accrued Coupon	$Fr_t = t \cdot Fr = .5(2.5) = 1.25$
Quoted Price	$price_t = 94.5098 - 1.25 = \mathbf{93.2598}$

(d) A 8.75-year 100, 5.5% bond yielding 6%

Price at date of previous coupon	$P_0 = 100v_{.03}^{18} + 2.75a_{\overline{18}\rceil.03} = 96.5616$
Time since previous coupon	$t = (.25 \text{ years})(2 \text{ coupon periods/year})$
	$= .5$ coupon periods
Price-plus-accrued	$P_t = P_0(1+j)^t = 96.5616(1.03)^{.5} = \mathbf{97.9993}$
Accrued Coupon	$Fr_t = t \cdot Fr = .5(2.75) = 1.375$
Quoted Price	$price_t = 97.9993 - 1.375 = \mathbf{96.6243}$

3. Consider a 10-year zero-coupon bond with a face value of \$1,000. Assume the effective annual interest rate is initially 8%. To what would the interest rate have to instantaneously increase in order for this bond to decrease in value by 25%?

$$P = \frac{1000}{(1.08)^{10}}$$

$$.75P = \frac{1000}{(1+i)^{10}}$$

$$(.75)\frac{1000}{(1.08)^{10}} = \frac{1000}{(1+i)^{10}}$$

$$\frac{(.75)^{1/10}}{1.08} = \frac{1}{1+i}$$

$$i = \frac{1.08}{(.75)^{1/10}} - 1 = \mathbf{11.15}\%$$

4. Consider a three-year bond, with a \$1,000 face value and a 9% coupon rate paid semi-annually, which was bought to yield 7% convertible semi-annually. Find the amount

of amortization of premium during the bonds third half-year (i.e., between the second and third coupon payments).

$$\begin{aligned} Pr_3 &= F(r-j)v_j^{n-k+1} \\ &= 1000(.045-.035)v_{.035}^4 \\ &= \mathbf{8.7144} \end{aligned}$$

5. A 30-year 6% semi-annual coupon bond with a face value of \$1,000 has a price of \$925. Find this bonds annual yield to maturity.

   BA II PLUS TVM Worksheet:

Keystrokes	*Display*	
[2ND] [CLR TVM]		
60 [N]	N=	60
925 [+|-] [PV]	PV=	-925
30 [PMT]	PMT=	30
1000 [FV]	FV=	1000
[CPT] [I/Y]	I/Y=	3.2879

$$i^{(2)} = (.032879)2 = \mathbf{6.58}\%$$

6. A 10-year 7% annual coupon bond has a face value of \$1,000. When you originally purchased this bond, the effective annual interest rate was 8%. Suppose that five years after purchase, the effective annual interest rate is 5%. What is the difference between the book and market values of the bond five years after purchase?

$$\begin{aligned} BV_5 &= 1000v_{.08}^5 + 70a_{\overline{5}|.08} = 960.073 \\ MV_5 &= 1000v_{.05}^5 + 70a_{\overline{5}|.05} = 1086.590 \end{aligned}$$

   The market value is $1086.590 - 960.073 = \mathbf{126.51}$ greater then the book value.

7. A 15-year 10% annual coupon bond has a face value of \$1,000. When you originally purchased this bond, the effective annual interest rate was 9%. Determine the proportion of the original bond premium which will have been amortized during the first five years after purchase.

$$\begin{aligned} BV_0 &= 1000 + 1000(.1-.09)a_{\overline{15}|.09} = 1080.61 \\ BV_5 &= 1000 + 1000(.1-.09)a_{\overline{10}|.09} = 1064.18 \\ Premium &= BV_0 - 1000 = 80.61 \end{aligned}$$

$$\begin{aligned} Premium\ Repaid &= BV_0 - BV_5 = 16.43 \\ Proportion\ of\ Bond\ Repaid &= \frac{BV_0 - BV_5}{BV_0} = 1.52\% \\ Proportion\ of\ Premium\ Amortized &= \frac{BV_0 - BV_5}{80.61} = 20.38\% \end{aligned}$$

8. A 15-year 8% bond has semi-annual coupons and a face amount of 100. It is quoted at a purchase price of 80. Find the yield rate.

   BA II PLUS Bond Worksheet:

Keystrokes	*Display*	
2ND [BOND] 2ND [CLR WORK]		
1.1001 ENTER	SDT=	1-10-2001
↓ 8 ENTER	CPN=	8
↓ 1.1016 ENTER	RDT=	1-10-2016
↓ 100 ENTER	RV=	100
↓ 2ND [SET] (if necessary)	360	
↓ 2ND [SET] (if necessary)	2/Y	
↓ ↓ 80	PRI=	80
↑ CPT	YLD=	**10.71**

9. Suppose the bond from question 8 was issued January 10, 2001 and was sold to a new investor for a *quoted* price of 105 on June 17, 2005. Find the yield rate for the new investor.

   BA II PLUS Bond Worksheet:

Keystrokes	*Display*	
6.1705 ENTER	SDT=	6-17-2005
↓ 8 ENTER	CPN=	8
↓ 1.1016 ENTER	RDT=	1-10-2016
↓ 100 ENTER	RV=	100
↓ 2ND [SET] (if necessary)	ACT	
↓ 2ND [SET] (if necessary)	2/Y	
↓ ↓ 105	PRI=	105
↑ CPT	YLD=	**7.31**

10. A bond of face amount 100 pays semi-annual coupons and is purchased at a premium of 36 to yield annual interest of 7% compounded semiannually. The amount of amortization of premium in the 5^{th} coupon is 1.00. What is the term of the bond?

$$\begin{aligned} 1 &= F(r-j)v_j^{n-k+1} \\ &= 100(r-.035)v_{.035}^{n-4} \\ &= Xv_{.035}^{n-4} \end{aligned}$$

$$v_{.035}^4 = Xv_{.035}^n$$

$$v_{.035}^n = \frac{v_{.035}^4}{X}$$

$$\begin{aligned}
36 &= F(r-j)a_{\overline{n}|j} \\
&= 100(r-.035)a_{\overline{n}|.035} \\
&= Xa_{\overline{n}|.035} \\
&= X\left(\frac{1-v_{.035}^{n}}{.035}\right) \\
&= X\left(\frac{1-\frac{v_{.035}^{4}}{X}}{.035}\right) \\
&= \frac{X-v_{.035}^{4}}{.035}
\end{aligned}$$

$$\begin{aligned}
X &= 36(.035)+v_{.035}^{4} \\
&= 2.13144
\end{aligned}$$

$$36 = 2.13144a_{\overline{n}|.035} \quad \Rightarrow \quad n = 25.99905425$$

The term is **13 years or 26 coupons**.