Math 231 A

Review Problems

(Solutions will be posted on the website on 12/12/06)

1. Evaluate each of the following integrals

$$(a) \int \frac{2^{1-\sqrt{x}}}{\sqrt{x}}\,dx; \quad (b) \int \sin^2 x \cos 3x\,dx; \quad (c) \int x^2(\ln(x))^2\,dx.$$

2. Determine whether the following improper integrals converge, and if so, find the value:

$$(a) \int_0^\infty \frac{x}{(1+x^2)^2}; \quad (b) \int_0^3 \frac{dx}{x^2-1}.$$

3. Find the area of one leaf of the polar curve with equation $r = \sin(3\theta)$.

4. A parametric curve has equations $x = t^3$, $y = t^2 - 1$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, and give a sketch of the curve.

5. Find the arc length of the parametric curve with equations $x = t^2$, $y = t^3$ from $t = 0$ to $t = 1$.

6. Find the surface area of revolution about the x-axis of that part of curve $y = x^3$ where $0 \le x \le 1$.

7. Test each of the following series for convergence or divergence:

$$(a)\ \sum_{n=2}^{\infty} \frac{1}{n(\ln(n))}; \quad (b)\ \sum_{n=2}^{\infty} \frac{1}{(\ln(n))^2}; \quad (c)\ \sum_{n=1}^{\infty} \frac{n(n+1)2^n}{(n-1)!}.$$

8. Determine whether each of the following series converges absolutely, converges conditionally or diverges:

$$(a)\ \sum_{n=1}^{\infty} (-1)^n \left(\frac{n\sqrt{n}-1}{n\sqrt{n}+1}\right); \quad (b)\ \sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{n}}{1+n}.$$

9. Find the first three *non-zero* terms of the Maclaurin series for the function $f(x) = \tan(x)$.

10. A non-elementary function $f(x)$ is defined by

$$f(x) = \int_0^x \frac{e^t - 1}{t}\, dt.$$

Find a power series in x for $f(x)$, including the nth term, by starting with the power series for e^t. Find also the radius of convergence of the series.