Homework II: June 20, 2011

Page 24, **Ex**. **6**. Solve using Gauss-Jordan reduction.

(a)

The augmented matrix of the system is

$$\begin{bmatrix} 1 & 1 & -1 \\ 4 & -3 & 3 \end{bmatrix}.$$

A reduced row-echelon form arising from this matrix is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}.$$

There is a unique solution: $(0, -1)$.

(d)

The augmented matrix of the system is

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & -1 & 3 & 0 \\ 1 & -2 & 1 & 1 & 0 \end{bmatrix}.$$

A reduced row-echelon form arising from this matrix is

$$\begin{bmatrix} 1 & 0 & 0 & \frac{4}{3} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \end{bmatrix}.$$

There are three lead variables: x_1, x_2, x_3 and one free variable: x_4. Putting the free variable on the right hand side gives the system

$$\begin{aligned} x_1 \qquad\qquad &= -\tfrac{4}{3}x_4 \\ x_2 \qquad &= -x_4 \\ x_3 &= \tfrac{1}{3}x_4. \end{aligned}$$

If we choose some value for x_4, say, $x_4 = \alpha$, then a solution is $(-\frac{4}{3}\alpha, 0, \frac{1}{3}\alpha, \alpha)$. Since α is arbitrary, every solution looks like

$$\alpha(-\tfrac{4}{3}, 0, \tfrac{1}{3}, 1).$$

Page 25, **Ex**. **10**. Consider a linear system whose augmented matrix is of the form

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{bmatrix}.$$

(a) For what values of a and b will the system have infinitely many solutions?

A row-echelon form for the system is

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-5 & b-4 \end{bmatrix}.$$

There will be infinitely many solutions if there is a free variable, and this can only happen here is the the bottom row is $[0 \quad 0 \quad u \quad v]$, where $u \neq 0$ and $v = 0$. Thus, $a \neq 5$ and $b = 4$.

(b) For what values of a and b will the system be inconsistent?

This can only happen here is the the bottom row is $[0 \quad 0 \quad u \quad v]$, where $u = 0$ and $v \neq 0$. Thus, $a = 5$ and $b \neq 4$.