Sixth Homework Set — Solutions
Chapter 4

Problem 35 Let X be the win/loss after one game. Then $P\{X=1.1\}=\frac{2\binom{5}{2}}{\binom{10}{2}}=\frac{20}{45}=\frac{4}{9}$, and $P\{X=-1\}=\frac{5}{9}$.

(a) $E[X]=1.1\cdot\frac{4}{9}-\frac{5}{9}=-\frac{1}{15}$.

(b) $\mathrm{Var}(X)=E[X^2]-E[X]^2=1.21\cdot\frac{4}{9}+\frac{5}{9}-\frac{1}{225}=1.0889$.

Problem 37

$$\begin{aligned}\mathrm{Var}(X)&=E\left[X^2\right]-E[X]^2\\&=\frac{40^3+33^2+25^3+50^3}{148}-\left(\frac{40^2+33^2+25^2+50^2}{148}\right)^2=82.2\\\mathrm{Var}(Y)&=\frac{40^2+33^2+25^2+50^2}{4}-37^2=84.5\end{aligned}$$

Problem 38 Note that $E[X^2]=\mathrm{Var}(X)+E[X]^2=5+1=6$.

(a) $E[(2+X)^2]=E[4+4X+X^2]=4+4E[X]+E[X^2]=14$.

(b) $\mathrm{Var}(4+3X)=9\mathrm{Var}(X)=45$.

Problem 40 Let X be the number of correct answers. Then

$$P\{X\geq 4\}=P\{X=4\}+P\{X=5\}=\binom{5}{4}\frac{1}{3^4}\cdot\frac{2}{3}+\frac{1}{3^5}=\frac{11}{243}.$$

Problem 42 See part (a) of Example 6f in the book.

Problem 45 Let A be the event that the student has an 'on' day, and let E_3, E_5 be the event that a majority of a panel of three (resp. five) examiners passes him. Then

$$
\begin{aligned}
P(A) &= \frac{1}{3}, P(A^c) = \frac{2}{3} \\
P(E_3|A) &= \binom{3}{2} 0.8^2 \cdot 0.2 + 0.8^3 = 0.896 \\
P(E_3|A^c) &= \binom{3}{2} 0.4^2 \cdot 0.6 + 0.4^3 = 0.352 \\
P(E_5|A) &= \binom{5}{3} 0.8^3 \cdot 0.2^2 + \binom{5}{4} 0.8^4 \cdot 0.2 + 0.8^5 = 0.9421 \\
P(E_5|A^c) &= \binom{5}{3} 0.4^3 \cdot 0.6^2 + \binom{5}{4} 0.4^4 \cdot 0.6 + 0.4^5 = 0.3174 \\
P(E_3) &= P(E_3|A) P(A) + P(E_3|A^c) P(A^c) = 0.5333 \\
P(E_5) &= P(E_5|A) P(A) + P(E_5|A^c) P(A^c) = 0.5256
\end{aligned}
$$

The student would be marginally better off with three examiners.

Problem 48 Let p be the probability that a single package contains more than one defective diskette. Then $p = 1 - 0.99^{10} - 10 \cdot 0.99^9 \cdot 0.01 = 0.0043$, and the probability of returning exactly one of three packages is $\binom{3}{1} p(1-p)^2 = 0.0127$.

Problem 50 Let F be the event that six of the first ten coin tosses come up heads.

(a) $P(H,T,T|E) = \frac{P(H,T,T \text{ and } E)}{P(E)} = \frac{p(1-p)^2 \binom{7}{5} p^5 (1-p)^2}{\binom{10}{6} p^6 (1-p)^4} = \frac{\binom{7}{5}}{\binom{10}{6}} = \frac{1}{10}$

(b) $P(T,H,T|E) = P(H,T,T|E) = \frac{1}{10}$