

## Seventh Homework Set — Solutions

### Chapter 4

Problem 55

$$\begin{aligned}
 P(\text{no errors}) &= P(\text{no errors}|\text{first typist})P(\text{first typist}) \\
 &\quad + P(\text{no errors}|\text{second typist})P(\text{second typist}) \\
 &= \frac{1}{2} \left( \frac{3^0}{0!}e^{-3} + \frac{4.2^0}{0!}e^{-4.2} \right) \\
 &= \frac{1}{2} (e^{-3} + e^{-4.2}).
 \end{aligned}$$

Problem 57  $X$  is Poisson with parameter  $\lambda = 3$ .

$$\begin{aligned}
 \text{(a)} \quad P\{X \geq 3\} &= 1 - P\{0\} - P\{1\} - P\{2\} = 1 - e^{-3} \left( 1 + 3 + \frac{9}{2} \right) = \\
 &= 0.5768. \\
 \text{(b)} \quad P\{X \geq 3|X \geq 1\} &= \frac{P\{X \geq 3\}}{P\{X \geq 1\}} = \frac{P\{X \geq 3\}}{1 - e^{-3}} = 0.6070.
 \end{aligned}$$

Problem 59 Let  $X$  be the number of times you win a prize. Then  $X$  is binomial with  $n = 50$  and  $p = \frac{1}{100}$ , i.e., we can use the Poisson approximation with  $\lambda = 50 \cdot \frac{1}{100} = \frac{1}{2}$ .

$$\begin{aligned}
 \text{(a)} \quad P\{X \geq 1\} &= 1 - P\{X = 0\} = 1 - e^{-\frac{1}{2}} = 0.3935 \\
 \text{(b)} \quad P\{X = 1\} &= \frac{1}{2}e^{-\frac{1}{2}} = 0.3033 \\
 \text{(c)} \quad P\{X \geq 2\} &= 1 - P\{X = 0\} - P\{X = 1\} = 1 - e^{-\frac{1}{2}} \left( 1 + \frac{1}{2} \right) = \\
 &= 0.0902
 \end{aligned}$$

Problem 61 Let  $X$  be Poisson with parameter  $\lambda = 1000 \cdot 0.0014 = 1.4$ . Then  $P\{X \geq 2\} = 1 - P\{X = 0\} - P\{X = 1\} = 1 - e^{-1.4}(1 + 1.4) = 0.4082$ .

Problem 63 Let  $X$  be a Poisson random variable with parameter  $\lambda = \frac{5}{2}$ . Then  $X$  gives a reasonable description of the number of people entering the casino between 12 and 12:05.

$$\begin{aligned}
 \text{(a)} \quad P\{X = 0\} &= e^{-\frac{5}{2}} = 0.0821 \\
 \text{(b)} \quad P\{X \geq 4\} &= 1 - e^{-\frac{5}{2}} \left( 1 + \frac{5}{2} + \frac{25}{8} + \frac{125}{48} \right) = 0.2424
 \end{aligned}$$