# 7th Homework Set — Solutions
## Chapter 5

Problem 25 Let $X$ be a binomial random variable with $p = 0.05$ and $n = 150$. Then $P\{X \leq 10\} = P\{X \leq 10.5\} = P\left\{\frac{X-7.5}{\sqrt{7.125}} \leq \frac{10.5-7.5}{\sqrt{7.125}}\right\} = \Phi(1.1239) = 0.8695$, using DeMoivre-Laplace.

Problem 28 Let $X$ be the number of lefthanders. Then $X$ is binomial with $p = 0.12$ and $n = 200$. Then

$$
\begin{aligned}
&P\{X \geq 20\} = P\{X > 19.5\} \\
&= P\left\{\frac{X-24}{\sqrt{200 \cdot 0.12 \cdot 0.88}} > \frac{19.5-24}{\sqrt{200 \cdot 0.12 \cdot 0.88}}\right\} \\
&= 1 - \Phi(-0.9792) = \Phi(0.9792) = 0.8363.
\end{aligned}
$$

Problem 32 Let $X$ be exponential with parameter $\lambda = \frac{1}{2}$.

(a) $P\{X > 2\} = 1 - F(2) = e^{-1}$

(b) $P\{X > 10 | X > 9\} = P\{X > 1\} = 1 - F(1) = e^{-\frac{1}{2}}$ because $X$ is memoryless.

Problem 33 Let $X$ be an exponential random variable with parameter $\lambda = \frac{1}{8}$. Since $X$ is memoryless, we have $P\{X > t + 8 | X > t\} = P\{X > 8\} = e^{-1}$.

Problem 34 Let $X$ be an exponential random variable with parameter $\lambda = \frac{1}{20}$. Since $X$ is memoryless, we have $P\{X > 30 | X > 10\} = P\{X > 20\} = e^{-1}$.

Let $Y$ be a uniform random variable on $[0, 40]$. Then $P\{X > 30 | X > 10\} = \frac{P\{X>30\}}{P\{X>10\}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$.

Problem 37 Let $X$ be uniformly distributed over $(-1, 1)$.

(a) $P\left\{|X| > \frac{1}{2}\right\} = P\left\{X > \frac{1}{2}\right\} + P\left\{X < -\frac{1}{2}\right\} = \frac{1}{2}$

(b) Let $Y = |X|$. If $y \in (0, 1)$, then $F_Y(y) = P\{Y \leq y\} = P\{-y \leq Y \leq y\} = y$, so that

$$
f_Y(y) = \begin{cases} 1 & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}
$$

Problem 39 Let $X$ be exponential with $\lambda = 1$, and let $Y = \log X$. Then $F_Y(y) = P\{Y \le y\} = P\{\log X \le y\} = P\{X \le e^y\} = 1 - e^{-e^y}$, so that

$$f_Y(y) = e^{y - e^y}.$$

Problem 40 Let $X$ be uniform on $(0,1)$, and $Y = e^X$. Then, for $1 < y < e$, $F_Y(y) = P\{Y \le y\} = P\left\{e^X \le y\right\} = P\{X \le \log Y\} = \log Y$, so that

$$f_Y(y) = \begin{cases} \frac{1}{y} & 1 < y < e \\ 0 & \text{otherwise} \end{cases}$$

Problem 41 For any $r \in (-A, A)$, we have $F_R r = P\{R \le r\} = P\{A \sin\theta \le r\} = P\left\{\theta \le \arcsin\frac{r}{A}\right\} = \frac{1}{\pi}\arcsin\frac{r}{A}$, so that

$$f_R(r) = \begin{cases} \frac{1}{\pi\sqrt{A^2 - r^2}} & r \in (-A, A) \\ 0 & \text{otherwise} \end{cases}$$