

## Fourth Homework Set — Solutions

### Chapter 3

- Problem 57 (a)  $P(\text{original price after two days}) = \binom{2}{1}p(1-p) = 2p(1-p)$   
(b)  $P(\text{increase by one after three days}) = \binom{3}{2}p^2(1-p) = 3p^2(1-p)$   
(c)  $P(\text{increase on first day}|\text{increase by one after three days}) = \frac{p \cdot 2p(1-p)}{3p^2(1-p)} = \frac{2}{3}$

- Problem 59 (a)  $P(HHHH) = p^4$   
(b)  $P(THHH) = p^3(1-p)$   
(c) The pattern  $HHHH$  can *only* occur before  $THHH$  if the first four coin flips come up heads. Hence,  $P(THHH \text{ occurs before } HHHH) = 1 - p^4$ .

Problem 64 Let  $E$  be the event that the wife answers correctly, and let  $F$  be the event that the husband answers correctly.

- (a) If only one of them answers, then the probability of a correct answer is  $P(E) = P(F) = p$ .  
(b)  $P(\text{correct answer}) = P(EF) + \frac{1}{2} \cdot 2 \cdot p(1-p) = p^2 + p - p^2 = p$

Problem 66 Let  $E_i$  be the event that the  $i$ -th switch is on.

- (a)

$$\begin{aligned} &P(\text{current flows from } A \text{ to } B) \\ &= (P(E_1E_2) + P(E_3E_4) - P(E_1E_2E_3E_4))P(E_5) \\ &= (p_1p_2 + p_3p_4 - p_1p_2p_3p_4)p_5 \end{aligned}$$

(b)

$$\begin{aligned} & P(\text{current flows from } A \text{ to } B) \\ &= P(E_1 E_4 \cup E_1 E_3 E_5 \cup E_2 E_5 \cup E_2 E_3 E_4) \\ &= p_1 p_4 + p_1 p_3 p_5 + p_2 p_5 + p_2 p_3 p_4 \\ &\quad - p_1 p_3 p_4 p_5 - p_1 p_2 p_4 p_5 - p_1 p_2 p_3 p_4 \\ &\quad - p_1 p_2 p_3 p_5 - p_1 p_2 p_3 p_4 p_5 - p_2 p_3 p_4 p_5 \\ &\quad + 4 p_1 p_2 p_3 p_4 p_5 - p_1 p_2 p_3 p_4 p_5 \\ &= p_1 p_4 + p_1 p_3 p_5 + p_2 p_5 + p_2 p_3 p_4 \\ &\quad - p_1 p_3 p_4 p_5 - p_1 p_2 p_4 p_5 - p_1 p_2 p_3 p_4 \\ &\quad - p_1 p_2 p_3 p_5 - p_2 p_3 p_4 p_5 + 2 p_1 p_2 p_3 p_4 p_5 \end{aligned}$$

Problem 78 (a)  $P(\text{exactly four games are played}) = P(ABAA) + P(BAAA) + P(ABBB) + P(BABB) = 2p^3(1-p) + 2p(1-p)^3 = 2p(1-p)(p^2 + (1-p)^2) = 2p(1-p)(1-2p+2p^2)$

(b) Let  $E$  be the event that  $A$  wins the match. Conditioning on the first two games of the match, we get  $P(E) = P(E|A, A)p^2 + P(E|A, B)p(1-p) + P(E|B, A)(1-p)p + P(E|B, B)(1-p)^2 = p^2 + 2P(E)p(1-p)$  because  $P(E|A, B) = P(E|B, A) = P(E)$ . Hence,  $P(E) = \frac{p^2}{1-2p(1-p)}$ .

Problem 81 Using the gambler's ruin formula, the answer is

$$\frac{1 - (9/11)^{15}}{1 - (9/11)^{30}}.$$

Problem 83 (a) Conditioning on the coin flip

$$P(\text{throw } n \text{ is red}) = \frac{1}{2} \frac{4}{6} + \frac{1}{2} \frac{2}{6} = \frac{1}{2}.$$

(b)

$$P(R_3 | R_1 R_2) = \frac{P(R_1 R_2 R_3)}{P(R_1 R_2)} = \frac{\frac{1}{2}(\frac{2}{3})^3 + \frac{1}{2}(\frac{1}{3})^3}{\frac{1}{2}(\frac{2}{3})^2 + \frac{1}{2}(\frac{1}{3})^2} = \frac{3}{5}.$$

(c)

$$P(A|R_1R_2) = \frac{P(R_1R_2|A)P(A)}{P(R_1R_2)} = \frac{\left(\frac{2}{3}\right)^{2\frac{1}{2}}}{\left(\frac{2}{3}\right)^{2\frac{1}{2}} + \left(\frac{1}{3}\right)^{2\frac{1}{2}}} = \frac{4}{5}.$$

Problem 84 (a)

$$\begin{aligned} P(A \text{ win}) &= \sum_{i=0}^{\infty} P(\text{ the first white appears on draw number } 3i + 1) \\ &= \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^{3i} \frac{1}{3} = \frac{1}{3} \sum_{i=0}^{\infty} \left(\frac{8}{27}\right)^i = \frac{1}{3} \frac{1}{1 - \frac{8}{27}} \end{aligned}$$

$$\begin{aligned} P(B \text{ win}) &= \sum_{i=0}^{\infty} P(\text{ the first white appears on draw number } 3i + 2) \\ &= \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^{3i+1} \frac{1}{3} = \frac{2}{9} \sum_{i=0}^{\infty} \left(\frac{8}{27}\right)^i = \frac{2}{9} \frac{1}{1 - \frac{8}{27}} \end{aligned}$$

$$\begin{aligned} P(C \text{ win}) &= \sum_{i=0}^{\infty} P(\text{ the first white appears on draw number } 3i + 3) \\ &= \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^{3i+2} \frac{1}{3} = \frac{4}{27} \sum_{i=0}^{\infty} \left(\frac{8}{27}\right)^i = \frac{4}{27} \frac{1}{1 - \frac{8}{27}} \end{aligned}$$

(b)

$$\begin{aligned} P(A \text{ win}) &= \frac{4}{12} + \frac{8}{12} \frac{7}{11} \frac{6}{10} \frac{4}{9} + \frac{8}{12} \frac{7}{11} \frac{6}{10} \frac{5}{9} \frac{4}{8} \frac{3}{7} \frac{4}{6} \\ P(B \text{ win}) &= \frac{8}{12} \frac{4}{11} + \frac{8}{12} \frac{7}{11} \frac{6}{10} \frac{5}{9} \frac{4}{8} + \frac{8}{12} \frac{7}{11} \frac{6}{10} \frac{5}{9} \frac{4}{8} \frac{3}{7} \frac{2}{6} \frac{4}{5} \\ P(C \text{ win}) &= \frac{8}{12} \frac{7}{11} \frac{4}{10} + \frac{8}{12} \frac{7}{11} \frac{6}{10} \frac{5}{9} \frac{4}{8} \frac{4}{7} + \frac{8}{12} \frac{7}{11} \frac{6}{10} \frac{5}{9} \frac{4}{8} \frac{3}{7} \frac{2}{6} \frac{1}{5}. \end{aligned}$$

### Chapter 3

Problem 1 Possible values of  $X$ : 0,2,4,-1,-2,1 Probabilities:

$$P\{X = 0\} = \frac{\binom{2}{2}}{\binom{14}{2}} = \frac{1}{91}$$

$$P\{X = 2\} = \frac{\binom{4}{1}\binom{2}{1}}{\binom{14}{2}} = \frac{8}{91}$$

$$P\{X = 4\} = \frac{\binom{4}{2}}{\binom{14}{2}} = \frac{6}{91}$$

$$P\{X = -1\} = \frac{\binom{8}{1}\binom{2}{1}}{\binom{14}{2}} = \frac{16}{91}$$

$$P\{X = -2\} = \frac{\binom{8}{2}}{\binom{14}{2}} = \frac{28}{91}$$

$$P\{X = 1\} = \frac{\binom{8}{1}\binom{4}{1}}{\binom{14}{2}} = \frac{32}{91}$$

Problem 4

$$P\{X = 1\} = \frac{\binom{5}{1}9!}{10!} = \frac{1}{2}$$

$$P\{X = 2\} = \frac{\binom{5}{1}\binom{5}{1}8!}{10!} = \frac{5}{18}$$

$$P\{X = 3\} = \frac{\binom{5}{2}2!\binom{5}{1}7!}{10!} = \frac{5}{36}$$

$$P\{X = 4\} = \frac{\binom{5}{3}3!\binom{5}{1}6!}{10!} = \frac{5}{84}$$

$$P\{X = 5\} = \frac{\binom{5}{4}4!\binom{5}{1}5!}{10!} = \frac{5}{252}$$

$$P\{X = 6\} = \frac{\binom{5}{5}5!\binom{5}{1}4!}{10!} = \frac{1}{252}$$

$$P\{X = 7\} = P\{X = 8\} = P\{X = 9\} = P\{X = 10\} = 0$$

Problem 5 The possible values are  $n, n - 2, n - 4, \dots, -n + 4, -n + 2, -n$ .

Problem 13 Let  $X$  be the total dollar value of all sales. Then  $X$  can take the values 0, 500, 1000, 1500, 2000, and we have

$$P\{X = 0\} = 0.7 \cdot 0.4 = 0.28$$

$$P\{X = 500\} = \frac{1}{2}(0.3 \cdot 0.4 + 0.7 \cdot 0.6) = 0.27$$

$$P\{X = 1000\} = \frac{1}{2}(0.3 \cdot 0.4 + 0.7 \cdot 0.6) + \frac{1}{4}0.3 \cdot 0.6 = 0.315$$

$$P\{X = 1500\} = 2\frac{1}{4}0.3 \cdot 0.6 = 0.09$$

$$P\{X = 2000\} = \frac{1}{4}0.3 \cdot 0.6 = 0.045$$

Problem 14

$$P\{X = 0\} = \frac{0!}{2!} = \frac{1}{2}$$

$$P\{X = 1\} = \frac{1!}{3!} = \frac{1}{6}$$

$$P\{X = 2\} = \frac{2!}{4!} = \frac{1}{12}$$

$$P\{X = 3\} = \frac{3!}{5!} = \frac{1}{20}$$

$$P\{X = 4\} = \frac{4!}{5!} = \frac{1}{5}$$