Math 562, Homework 1

Due 09/25

1. Show that if X is a normal random variable with mean zero and variance σ^2, show that for each positive integer n, there is a positive constant C_n such that

$$E(X^{2n}) = C_n \sigma^{2n}.$$

Find the explicit value of C_n.

2. Suppose that $\{X^{(n)}\}_{n=1}^{\infty}$ is a sequence of real-valued continuous process $X^{(n)} = \{X_t^{(n)} : t \geq 0\}$ on some probability space $(\Omega, \mathcal{F}, P)$. Suppose that there exist positive constants ν, α, β such that

(i) $\sup_{n\geq 1} E\left(|X_0^{(n)}|^{\nu}\right) < \infty$;

(ii) for each $T > 0$, there is $C_T > 0$ such that $\sup_{n\geq 1} E\left(|X_t^{(n)} - X_s^{(n)}|^{\alpha}\right) \leq C_T |t-s|^{1+\beta}$ for all $0 \leq s, t \leq T$.

Let P_n be the probability measure on $(C[0,\infty), \mathcal{B}(C[0,\infty)))$ induced by $X^{(n)}$. Show that $\{P_n\}_{n=1}^{\infty}$ is a tight sequence.

3. Suppose that $\{P_n\}_{n=1}^{\infty}$ is a sequence of probability measures on $(C[0,\infty), \mathcal{B}(C[0,\infty)))$ which converge weakly to a probability measure P. Suppose that $\{f_n\}_{n=1}^{\infty}$ is a (uniformly) bounded sequence of continuous real-valued functions on $(C[0,\infty)$ converging to a continuous function f, the convergence being uniform on compact subsets of $(C[0,\infty)$. Show that

$$\lim_{n\to\infty} \int_{C[0,\infty)} f_n(\omega) P_n(d\omega) = \int_{C[0,\infty)} f(\omega) P(d\omega).$$