Math 562, Homework 2

Due 10/02

1. Prove that, for any $m \in \mathbf{R}^d$ and any symmetric nonnegative definite $d \times d$ matrix Σ, there is an $\mathbf{R}^d$-valued random variable ξ with mean vector m and covariance matrix Σ.

2. Let $(\mathcal{F}_t)$ be a filtration on $(\Omega, \mathcal{F})$ and S, T be $(\mathcal{F}_t)$-stopping times. Show that

(i) $S \wedge T$, $S \vee T$ and $S + T$ are $(\mathcal{F}_t)$-stopping times;

(ii) the sets $\{S = T\}$, $\{S \leq T\}$, $\{S < T\}$ are in $\mathcal{F}_S \cap \mathcal{F}_T$;

(iii) if $S \leq T$, then $\mathcal{F}_S \subset \mathcal{F}_T$.

3. Suppose that (T_n) is a sequence of $(\mathcal{F}_t)$-stopping times.

(i) Show that $\sup_n T_n$ is an $(\mathcal{F}_t)$-stopping time.

(ii) Furthermore, if $(\mathcal{F}_t)$ is right continuous, then $\inf_n T_n$, $\limsup_{n\to\infty} T_n$ and $\liminf_{n\to\infty} T_n$ are $(\mathcal{F}_t)$-stopping times. If $T_n \downarrow T$, then $\mathcal{F}_T = \cap_n \mathcal{F}_{T_n}$.