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Lévy risk model with two-sided jumps and a barrier dividend strategy

Lijun Bo[a], Renming Song[b], Dan Tang[c], Yongjin Wang[d], Xuewei Yang[e,b,*]

[a] Department of Mathematics, Xidian University, Xi'an 710071, PR China
[b] Department of Mathematics, University of Illinois, Urbana, IL 61801, USA
[c] School of International Trade and Economics, University of International Business and Economics, Beijing 100029, PR China
[d] School of Business, Nankai University, Tianjin 300071, PR China
[e] School of Mathematical Sciences, Nankai University, Tianjin 300071, PR China

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ABSTRACT

In this paper, we consider a general Lévy risk model with two-sided jumps and a constant dividend barrier. We connect the ruin problem of the ex-dividend risk process with the first passage problem of the Lévy process reflected at its running maximum. We prove that if the positive jumps of the risk model form a compound Poisson process and the remaining part is a spectrally negative Lévy process with unbounded variation, the Laplace transform (as a function of the initial surplus) of the upward entrance time of the reflected (at the running infimum) Lévy process exhibits the smooth pasting property at the reflecting barrier. When the surplus process is described by a double exponential jump diffusion in the absence of dividend payment, we derive some explicit expressions for the Laplace transform of the ruin time, the distribution of the deficit at ruin, and the total expected discounted dividends. Numerical experiments concerning the optimal barrier strategy are performed and new empirical findings are presented.

1. Introduction

Since the pioneering work by De Finetti (1957), the problem of finding the optimal dividend-payment strategy has been studied extensively. De Finetti found that, if the goal is to maximize the expected discounted dividends, the optimal strategy must be a barrier strategy. Some related works on this subject include, among others, Gerber and Shiu (1998, 2004), Siegl and Tichy (1999), Højgaard (2002), Irbäck (2003), Sheldon et al. (2003), Zhou (2005), Kyprianou and Palmowski (2007), Renaud and Zhou (2007), Belhaj (2010). Recently, based on the fluctuation theory of spectrally negative Lévy processes, Avram et al. (2007), Loeffen (2008), Loeffen (2009) have studied the optimal dividend problem for general spectrally negative Lévy processes, and provided sufficient conditions under which the barrier strategy solves the de Finetti optimal dividend problem. Kyprianou et al. (2010) further generalized the results in Avram et al. (2007) and Loeffen (2008, 2009) by showing that if the Lévy measure of the spectrally negative Lévy process has a log convex density, the barrier strategy is optimal. Loeffen and Renaud (2010) finally proved that the results on optimal dividend strategies of these papers are still valid when the corresponding condition imposed on the Lévy measure is replaced by the condition that the tail of the Lévy measure is

* Correspondence to: School of Mathematical Sciences, Nankai University, #94 Weijin Road, Nankai District, Tianjin 300071, PR China.
E-mail addresses: bolijunnk@yahoo.com.cn (L. Bo), rsong@math.uiuc.edu (R. Song), dantangcn@gmail.com (D. Tang), yjwang@nankai.edu.cn (Y. Wang), xwyangnk@yahoo.com.cn (X. Yang).

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log-convex. All of the above mentioned work is based on spectrally one-sided models.

Recently, risk models with two-sided jumps attract more and more attention. In this kind of models, the upward jumps can be interpreted as random returns (obtained by investing the initial asset and the insurance premium) of an insurance company, while the downward jumps are interpreted as random losses (from investment or claim indemnity) of the company. For research on this kind of models, we refer, among others, to Perry et al. (2002), Cai and Yang (2005), Jacobsen (2005), Xing et al. (2008), Cai et al. (2009), Zhang et al. (2010), Chi (2010) and Albrecher et al. (2010). Most recently, Chi and Lin (2011) studied the threshold dividend strategy when the risk process follows a Lévy process with two-sided jumps.

The current paper aims at studying risk models with two-sided jumps and a (constant) barrier dividend strategy. To the best of our knowledge, Paulsen and Gjessing (1997), Yin and Yuen (2011) and Yuen and Yin (2011) are the only papers that addressed the barrier dividend problem with two-sided jumps. However, the proofs in these three papers are questionable. It is worthwhile to note that when the surplus process can jump upward, the threshold dividend strategy is drastically different from the barrier dividend strategy, since the former generates a continuous dividend process, while the latter creates a discontinuous one. Thus the techniques used in Chi and Lin (2011) for studying the threshold dividend strategy are not feasible for the barrier dividend strategy in this paper.

The purpose of this paper is to establish some easily implementable results on a general Lévy risk model with two-sided jumps and a barrier dividend strategy. More specifically, we are going to show that the constant barrier dividend problem can be explicitly solved as well for some spectrally two-sided Lévy processes. We first relate the ruin problem of the risk model with barrier dividend strategy to the first passage problem of the risk model reflected at its running maximum (see Proposition 2.1). For a general Lévy risk model, we show that the expected discounted dividend can be expressed in terms of the Laplace transform of the upward entrance time of the unconstrained Lévy process and the joint Laplace transform of the upward entrance time and the overshoot of the Lévy process reflected at its running maximum (see Theorem 2.1). If the Lévy risk model can be decomposed into two parts, namely a spectrally negative Lévy process with unbounded variation and a subordinator, we can prove that the Laplace transform (as a function of the initial surplus) of the upward entrance time of the Lévy process reflected at its running infimum possesses the smooth pasting property at the reflecting barrier 0. A more general version of this result is proved when the underlying risk model is the so-called double exponential jump diffusion (see, e.g. Kou and Wang, 2003, 2004). The smooth pasting property will serve as a homogeneous Neumann boundary condition when we solve the Feynman–Kac integro-differential equations corresponding to the first passage problems of the reflected Lévy processes. We then study the (joint) Laplace transform of the upward entrance time and the overshoot for the double exponential jump diffusion reflected at its running infimum and maximum, respectively. Then, applying our results above, we find some explicit expressions for the Laplace transform of the time of ruin, the distribution of the deficit at ruin and the expected discounted dividends up to ruin. All our results on the ruin problem are expressed in terms of the parameters of the jump size and the solutions to the Cramér–Lundberg equation corresponding to the underlying double exponential jump diffusion or its dual. A nice feature of our results is that they are explicit functions of the initial surplus and the barrier parameter, which is very handy when we want to solve the optimal dividend barrier. Finally, we present some numerical experiments related to the optimal barrier strategy. The most important empirical finding is that the optimal dividend barrier would depend on the initial surplus if the initial surplus is less than some critical value (in our experiment, the optimal barrier decreases to a positive value as the initial surplus decreases to zero); whereas if the initial surplus is greater than or equal to the critical value, the optimal dividend barrier will be equal to the critical value. The dependence on the initial surplus of the optimal dividend barrier is different from the case in the spectrally negative setting (see, e.g., Gerber and Shiu (1998, 2004) and Kyprianou et al. (2010)), which is due to the incorporation of two-sided jumps in the risk model. Note that a barrier strategy that depends on the initial surplus cannot be optimal among all admissible dividend strategies.

The dividend process corresponding to the barrier strategy is exactly the so-called regulator (or the local time) at the dividend barrier, which can be given as the solution to a Skorokhod problem (see, e.g., Skorokhod (1961), Harrison (1985), Doney and Maller (2007) and Asmussen and Pihlsgard (2007)). The post-dividend surplus process is the so-called reflected jump-diffusion (or jump-diffusion with reflecting barrier). Some theoretical results related to reflected spectrally one-sided Lévy processes can be found in Pistorius (2003, 2004). Kella and Whitt (1992) and Nguyen-Ngoc and Yor (2005) introduced some useful martingales related to the reflected Lévy processes, which are very powerful in various applications in queueing, finance and insurance theory.

Our paper is organized as follows. Section 2 investigates the general Lévy risk model. In particular, we will establish some key identities between the ruin problem and the first passage problem of a Lévy process reflected at its running maximum. In Section 3, we concentrate on the double exponential jump diffusion as a solvable example. Therein we will derive some explicit expressions for the Laplace transform of the time of ruin, the distribution of the deficit at ruin, and the expected discounted dividends. Section 4 provides numerical results on optimal dividend strategy. Section 5 concludes the paper and discusses some potential further research. Some proofs are given in the Appendix.

2. General Lévy risk model with two-sided jumps

Let $X = \{X_t, t \geq 0\}$ be a Lévy process on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$, where $\{\mathcal{F}_t\}_{t\geq 0}$ satisfies the usual conditions of right-continuity and completeness. Let σ and Π be the Gaussian coefficient and the Lévy measure of X respectively. When X is of bounded variation, we will write $X_t = dt + J_t$ where d is the drift and J is a pure jump Lévy process. Throughout this paper, we assume that either X has unbounded variation or Π is absolutely continuous with respect to the Lebesgue measure, i.e.,

$$\sigma > 0 \quad \text{or} \int_{|x|<1} |x|\Pi(dx) = \infty \text{ or } \Pi(dx) \ll dx. \tag{2.1}$$

Denote by $\mathbb{P}_x, x \in \mathbb{R}$ the law of $X + x$ under $\mathbb{P}$. Let $\mathbb{E}_x$ be the expectation operator corresponding to $\mathbb{P}_x$. Let T_a^+ and T_a^- be the entrance times of the Lévy process X into $(a, +\infty)$ and $(-\infty, -a)$, respectively:

$$T_a^+ = \inf\{t \geq 0: X_t > a\}, \qquad T_a^- = \inf\{t \geq 0: X_t < -a\}, \tag{2.2}$$

with the convention $\inf \emptyset = \infty$. Define $Y := X - I$ and $\hat{Y} = S - X$ as the Lévy process X reflected at its running infimum I and at its running supremum S, respectively:

$$I_t := \inf_{s\leq t}(X_s \wedge 0), \qquad S_t = \sup_{s\leq t}(X_s \vee 0). \tag{2.3}$$

Let $\hat{X} = -X$ be the dual process of X, we have

$$\begin{aligned} S_t &= \sup_{0\leq s\leq t}(X_s \vee 0) = -\inf_{0\leq s\leq t}((-X_s) \wedge 0) \\ &= -\inf_{0\leq s\leq t}(\hat{X}_s \wedge 0) =: -\hat{I}, \end{aligned} \tag{2.4}$$

and thus the process $\hat{Y} = S - X = \hat{X} - \hat{I}$ is the dual process $\hat{X}$ reflected at its running infimum. Further, denote by τ_a (resp. $\hat{\tau}_a$) the entrance time of the reflected process Y (resp. $\hat{Y}$) into (a, ∞):

$$\tau_a = \inf\{t \geq 0: Y_t > a\}, \qquad \hat{\tau}_a = \inf\{t \geq 0: \hat{Y}_t > a\}. \tag{2.5}$$

Note that the reflected Lévy processes Y and $\hat{Y}$ are Markov processes.

2.1. Risk model with constant barrier dividend strategy

In this subsection, we consider a risk model with constant barrier dividend strategy. Let X be the risk process of an insurance company before dividends are deduced. The company will pay dividends according to a barrier strategy with parameter $b > 0$. Define the running maximum by $M_t = \sup_{0\leq s\leq t} X_s$, then the aggregate dividends paid by time t are

$$L_t^b = \max\{M_t - b, 0\} = \sup_{s\leq t}[X_s - b] \vee 0. \tag{2.6}$$

Throughout this paper, we use U^b to denote the risk process regulated by the dividend payment L^b, that is

$$U_t^b = X_t - L_t^b \leq b, \quad t \geq 0. \tag{2.7}$$

Roughly speaking, the dividend process $(L_t^b)_{t\geq 0}$ is the magnitude of the displacement which is the minimal amount required to keep $(U_t^b)_{t\geq 0}$ always less than or equal to b. Moreover, the dividend process $(L_t^b)_{t\geq 0}$ has the following properties (see, e.g., Harrison (1985), Asmussen and Pihlsgard (2007) and Andersen and Asmussen (2009)):

(1) the paths of $t \to L_t^b$ have càdlàg modifications, nondecreasing and $L_0^b = \max\{X_0 - b, 0\}$;

(2) for all $t \geq 0$,

$$L_t^b = \int_{[0,t]} \mathbb{1}_{\{U_u^b = b\}} dL_u^b. \tag{2.8}$$

Denote by $\tilde{\tau}_b$ the ruin time of the insurance company with surplus process U^b

$$\tilde{\tau}_b = \inf\{t \geq 0: U_t^b < 0\}.$$

Then we have the following results concerning the Laplace transform of the ruin time, the deficit at ruin, and the expected discounted dividends (see also Proposition 1 in Avram et al. (2007), p. 162):

Proposition 2.1. *Suppose $b > 0$ and $x \in [0, b]$. Then we have the following:*

(I) *The Laplace transform of the ruin time is given by*

$$\mathbb{E}_x\left[e^{-r\tilde{\tau}_b}\right] = \mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b}\right]. \tag{2.9}$$

(II) *The total expected discounted dividends before ruin satisfies*

$$\mathbb{E}_x\left[\int_0^{\tilde{\tau}_b} e^{-rt} dL_t^b\right] = \mathbb{E}_{x-b}\left[\int_0^{\hat{\tau}_b} e^{-rt} dS_t\right]. \tag{2.10}$$

(III) *The deficit at ruin satisfies, for each $y \geq 0$*

$$\mathbb{E}_x\left[e^{-r\tilde{\tau}_b}\mathbb{1}_{\{-U_{\tilde{\tau}_b}^b > y\}}\right] = \mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b}\mathbb{1}_{\{\hat{Y}_{\hat{\tau}_b} - b > y\}}\right]. \tag{2.11}$$

Moreover, for any bounded Borel function f, we have

$$\mathbb{E}_x\left[e^{-r\tilde{\tau}_b} f(-U_{\tilde{\tau}_b}^b)\right] = \mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b} f(\hat{Y}_{\hat{\tau}_b} - b)\right]. \tag{2.12}$$

Proof. By the spatial homogeneity of the surplus process X, it is not hard to see that $\{U^b, L^b, \tilde{\tau}_b; U_0 = x\}$ has the same law as $\{b - \hat{Y}, S, \hat{\tau}_b; \hat{Y}_0 = b - x\}$. Note that $\hat{Y}_0 = b - x$ if $X_0 = x - b$. The conclusion follows easily from this fact. □

We next present a useful expression for the total expected discounted dividends (see Theorem 1 in Avram et al. (2007) for the spectrally negative case).

Theorem 2.1. *For the Lévy process X, suppose 0 is regular for $(0, \infty)$.*[1] *Let $b > 0$. For $x \in [0, b]$, the total expected discounted dividends can be expressed as*

$$\mathbb{E}_x\left[\int_0^{\tilde{\tau}_b} e^{-rt} dL_t^b\right] = k(x - b) - \mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b} k(-\hat{Y}_{\hat{\tau}_b})\right], \tag{2.13}$$

where, for $x \geq 0$,

$$k(-x) = \mathbb{E}[e^{-rT_x^+}(X_{T_x^+} - x)] + \mathbb{E}[e^{-rT_x^+}] \int_0^\infty \mathbb{E}[e^{-rT_z^+}] dz. \tag{2.14}$$

Proof. From Proposition 2.1(II), we have

$$\begin{aligned}
\mathbb{E}_x\left[\int_0^{\tilde{\tau}_b} e^{-rt} dL_t^b\right] &= \mathbb{E}_{x-b}\left[\int_0^{\hat{\tau}_b} e^{-rt} dS_t\right] \\
&= \mathbb{E}_{x-b}\left[\int_0^\infty e^{-rt} dS_t\right] \\
&\quad - \mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b}\mathbb{E}_{-\hat{Y}_{\hat{\tau}_b}}\left(\int_0^\infty e^{-rt} dS_t\right)\right] \\
&:= k(x - b) - \mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b} k(-\hat{Y}_{\hat{\tau}_b})\right],
\end{aligned} \tag{2.15}$$

where, for $x \geq 0$,

$$\begin{aligned}
k(-x) &= \mathbb{E}_{-x}\left[\int_0^\infty e^{-rt} dS_t\right] \\
&= \mathbb{E}_{-x}[e^{-rT_0^+} X_{T_0^+}] + \mathbb{E}_{-x}[e^{-rT_0^+}]\mathbb{E}\left[\int_0^\infty e^{-rt} dS_t\right] \\
&= \mathbb{E}[e^{-rT_x^+}(X_{T_x^+} - x)] + \mathbb{E}[e^{-rT_x^+}] \int_0^\infty \mathbb{E}[S_t] r e^{-rt} dt \\
&= \mathbb{E}[e^{-rT_x^+}(X_{T_x^+} - x)] \\
&\quad + \mathbb{E}[e^{-rT_x^+}] \int_0^\infty r e^{-rt} dt \int_0^\infty \mathbb{P}(S_t \geq z) dz \\
&= \mathbb{E}[e^{-rT_x^+}(X_{T_x^+} - x)] \\
&\quad + \mathbb{E}[e^{-rT_x^+}] \int_0^\infty r e^{-rt} dt \int_0^\infty \mathbb{P}(T_z^+ \leq t) dz \\
&= \mathbb{E}[e^{-rT_x^+}(X_{T_x^+} - x)] + \mathbb{E}[e^{-rT_x^+}] \int_0^\infty \mathbb{E}[e^{-rT_z^+}] dz,
\end{aligned}$$

[1] 0 is regular for $(0, \infty)$ means that a Lévy process started at the origin gets to $(0, \infty)$ at arbitrarily small times. See Definition 6.4 in Kyprianou (2006), p. 142.

where the second equality holds since $\hat{Y}_{T_0^+} = 0$, and the penultimate one follows from the regularity of 0 for $(0, \infty)$. The proof is now complete. □

Remark 2.1. Theorem 2.1 shows that if we want to compute the expected discounted dividends when the underlying risk model is a general Lévy process X with two-sided jumps, we need to know the Laplace transform of the one-sided first (upward) passage time of X as well as the joint distribution of the (upward) entrance time and the overshoot of the reflected Lévy process $\hat{Y} = S - X$.

We recall some known results for spectrally negative Lévy processes in the next subsection, and then in Section 2.3 we will prove that under some appropriate conditions the Laplace transform (as a function of the initial value) of the upward entrance time of a general Lévy process (with two-sided jumps) reflected at its running infimum satisfies the smooth pasting property at the reflecting barrier 0. The smooth pasting property (which is a homogeneous Neumann boundary condition) will be used to derive the explicit solution for the ruin problem by solving a Feynman–Kac integro–differential equation in Section 3.

2.2. Spectrally negative Lévy processes

In this subsection we recall some known results for (reflected) spectrally negative Lévy processes and show that under some appropriate conditions the Laplace transform (as a function of initial value) of the upward entrance time of a spectrally negative Lévy process reflected at its running infimum satisfies the smooth pasting property at the reflecting barrier 0.

We rewrite X as

$$X_t = X_t^{(-)} + X_t^{(+)}, \tag{2.16}$$

where $X^{(-)}$ is a spectrally negative Lévy process, and $X^{(+)}$ is a pure jump Lévy process which can only jump upward. Denote the Laplace exponent of $X^{(-)}$ by $\psi^{(-)}$

$$\psi^{(-)}(\zeta) := \log \mathbb{E}\left[e^{\zeta X_1^{(-)}}\right], \tag{2.17}$$

which is well defined at least in the right half complex plane. Denote by $Y^{(-)} := X^{(-)} - I^{(-)}$ the spectrally negative Lévy process $X^{(-)}$ reflected at its running infimum

$$I_t^{(-)} := \inf_{s \le t}\left(X_s^{(-)} \wedge 0\right). \tag{2.18}$$

Let $\tau_a^{(-)}$ be the entrance time of the reflected process $Y^{(-)}$ into (a, ∞)

$$\tau_a^{(-)} = \inf\{t \ge 0 \colon Y_t^{(-)} > a\}. \tag{2.19}$$

It was shown in Pistorius (2004) that, for $x \in [0, a]$

$$g^{(-)}(x) := \mathbb{E}_x\left[e^{-r\tau_a^{(-)}}\right] = \frac{Z^{(r)}(x)}{Z^{(r)}(a)}, \tag{2.20}$$

where $Z^{(r)}(x) = 1 + r\int_0^x W^{(r)}(y)dy$ with $W^{(r)}(x)$ being the *r-scale* function, which is increasing and continuously differentiable (see, e.g., Lambert (2000)) on $(0, \infty)$ with Laplace transform

$$\int_0^\infty e^{-\zeta x} W^{(r)}(x)dx = \frac{1}{\psi^{(-)}(\zeta) - r}, \quad \zeta > \Phi^{(-)}(r), \tag{2.21}$$

where $\Phi^{(-)}(\cdot)$ is the inverse function of the Laplace exponent $\psi^{(-)}(\cdot)$. If $X^{(-)}$ is of bounded variation and $d^{(-)}$ is the drift of $X^{(-)}$, we have (see, e.g., Avram et al. (2004))

$$Z^{(r)\prime}(0+) = rW^{(r)}(0) = r/d^{(-)}. \tag{2.22}$$

When $X^{(-)}$ is of unbounded variation, it holds that (see, e.g., Lemma 8.3 and Exercise 8.5 in Kyprianou (2006))

$$Z^{(r)\prime}(0) = rW^{(r)}(0) = 0. \tag{2.23}$$

From the above results, we have the following

Proposition 2.2. *If the spectrally negative Lévy process $X^{(-)}$ is of unbounded variation, then*

$$g^{(-)\prime}(0) = 0. \tag{2.24}$$

If $X^{(-)}$ is of bounded variation, we have

$$0 = g^{(-)\prime}(0-) \neq g^{(-)\prime}(0+) = \frac{r}{dZ^{(r)}(a)}. \tag{2.25}$$

Proof. Note that $g^{(-)\prime}(x) = \frac{Z^{(r)\prime}(x)}{Z^{(r)}(a)}$ on $[0, a]$. It follows from (2.23) that $g^{(-)\prime}(0+) = 0$. On the other hand, by the definition of $Y^{(-)}$, we have $g^{(-)}(x) = g^{(-)}(0)$ for $x < 0$, which yields that $g^{(-)\prime}(0-) = 0$. We have proved the first conclusion. The other conclusion follows from (2.22). □

Remark 2.2. The above proposition shows that, when $X^{(-)}$ is a spectrally negative Lévy process with unbounded variation, the Laplace transform $g^{(-)}(x)$ defined in (2.20) satisfies the smooth pasting property at the reflecting barrier 0. This phenomenon has been well documented for the classical diffusion models (see, e.g., Chapter 15 in Karlin and Taylor (1981) and Linetsky (2005)). Next we will provide a sufficient condition for a general Lévy processes such that smooth pasting occurs at the reflecting barrier.

2.3. Smooth pasting at the reflecting barrier: general Lévy processes

In this subsection, we will prove that under some appropriate conditions, the Laplace transform of the upward entrance time of the Lévy process reflected at its running infimum has the smooth pasting property at the reflecting boundary 0, which will be needed to find the explicit solution for the ruin problem in Section 3. To this end, define the Laplace transform of the entrance time τ_a of the reflected Lévy process Y by

$$g(x) = \mathbb{E}_x\left[e^{-r\tau_a}\right], \quad a > 0, r > 0. \tag{2.26}$$

Then we have (recall the decomposition (2.16))

Lemma 2.1. *If $X^{(-)}$ is of unbounded variation and $X^{(+)}$ is a subordinator, then*

$$g'(0+) = 0. \tag{2.27}$$

Proof. Recall that $Y^{(-)} = X^{(-)} - I^{(-)}$ and $\tau_a^{(-)}$ is the entrance time of $Y^{(-)}$ into (a, ∞). We have, for $0 < x < a$,

$$\begin{aligned}\mathbb{E}_x\left[e^{-r\tau_a}\right] - \mathbb{E}\left[e^{-r\tau_a}\right] &\le \mathbb{E}\left[\mathbb{E}_{Y_{\tau_x}}\left[e^{-r\tau_a}\right]\right] - \mathbb{E}\left[e^{-r\tau_x}\mathbb{E}_{Y_{\tau_x}}\left[e^{-r\tau_a}\right]\right] \\ &\le \mathbb{E}\left[\left(1 - e^{-r\tau_x}\right)\right] \\ &\le \mathbb{E}\left[\left(1 - e^{-r\tau_x^{(-)}}\right)\right],\end{aligned}$$

where the last inequality follows from the fact that $\tau_x < \tau_x^{(-)}$.[2] Recall from (2.20) that[3]

$$\mathbb{E}[e^{-r\tau_x^{(-)}}] = \frac{1}{Z^{(r)}(x)} \to 1, \quad \text{as } x \downarrow 0.$$

Thus we have

$$\begin{aligned} 0 \le g'(0+) &= \lim_{x\downarrow 0} \frac{g(x)-g(0)}{x} \\ &= \lim_{x\downarrow 0} \frac{\mathbb{E}_x\left[e^{-r\tau_a}\right] - \mathbb{E}\left[e^{-r\tau_a}\right]}{x} \\ &\le \lim_{x\downarrow 0} \frac{1-\mathbb{E}\left[e^{-r\tau_x^{(-)}}\right]}{x} \\ &= 0, \end{aligned}$$

where the last equality follows by using L'Hôpital's rule and noting that when X is of unbounded variation $Z^{(q)\prime}(0) = W^{(q)}(0) = 0$. □

Conjecture. *The Laplace transform $g(x)$ defined in (2.26) has the smooth pasting property at the reflecting barrier 0 if X is of unbounded variation.*

Remark 2.3. As in the diffusion case, we impose a homogeneous Neumann boundary condition when the corresponding boundary is instantaneously reflecting, which says that the diffusion will leave the boundary immediately (with an infinite speed) after the diffusion hits the boundary. The above conjecture is based on the fact that when the Lévy process is of unbounded variation, it will also leave the boundary at an infinite speed as soon as it hits the reflecting barrier.

The following corollary of Lemma 2.1 provides an expression for the Laplace transform of the entrance time of the reflected Lévy process in terms of the Laplace transform of the two-sided exit time of the unconstrained Lévy process.

Corollary 2.1. *Let $a > 0$ and $x \in [0, a]$. If $X^{(-)}$ is of unbounded variation and $X^{(+)}$ is a compound Poisson process, then the Laplace transform of the entrance time of the reflected Lévy process Y is*

$$\mathbb{E}_x\left[e^{-r\tau_a}\right] = u_u(x) - \frac{u_u'(0+)}{u_d'(0+)} u_d(x), \quad r > 0, \tag{2.28}$$

provided $u_d'(0+) \neq 0$, where $u_u(x) = \mathbb{E}_x\left[e^{-rT_a^+}\mathbb{1}_{\{T_a^+ < T_0^-\}}\right]$ and $u_d(x) = \mathbb{E}_x\left[e^{-rT_0^-}\mathbb{1}_{\{T_a^+ > T_0^-\}}\right]$.

Proof. Note that $Y_{T_0^-} = 0$ (due to the reflection). From the strong Markov property, we have

$$\begin{aligned} f(x) := \mathbb{E}_x\left[e^{-r\tau_a}\right] &= \mathbb{E}_x\left[e^{-rT_a^+}\mathbb{1}_{\{T_a^+ < T_0^-\}}\right] \\ &\quad + \mathbb{E}_x\left[e^{-rT_0^-}\mathbb{1}_{\{T_a^+ > T_0^-\}}\right]\mathbb{E}\left[e^{-r\tau_a}\right] \\ &= u_u(x) + u_d(x) f(0). \end{aligned}$$

Taking derivative on both side of the above equation and letting x approach to zero yield the following equation for $f(0)$

$$0 = f'(0+) = u_u'(0+) + u_d'(0+) f(0),$$

thus $f(0) = -u_u'(0+)/u_d'(0+)$. The conclusion follows by substituting $f(0)$ back into the expression for $f(x)$. □

3. The double exponential jump-diffusion model

The double exponential jump diffusion is a special one-dimensional Lévy processes with two-sided jumps which have been studied in finance by many authors (see, e.g., Kou (2002), Kou and Wang (2003, 2004), Sepp (2004) and Ramezani and Zeng (2007)) due to its analytical tractability and its consistency with the asymmetric leptokurtosis of the return distribution and volatility smile in option pricing. In this section, we will show that the ruin problem (time of ruin, deficit at ruin and the expected discounted dividend) with barrier dividend strategy can be explicitly solved under the double exponential jump diffusion model.[4] We first study the first passage problem of the reflected double exponential jump diffusion, then provide explicit solutions to the ruin problem as corollaries of our results above. We here mention that Albrecher et al. (2010) studied this model without dividend barrier. Gerber and Yang (2010) investigated the dividends-penalty identity for models with two-sided jumps, while the authors did not deal with the risk model with a diffusion part. See also Albrecher et al. (2011) for some related calculations for this model.

3.1. (Reflected) double exponential jump diffusion

Throughout this section, we suppose that the surplus (in the absence of dividend payment) of an insurance company follows the following double exponential jump diffusion

$$X_t = \mu t + \sigma W_t + \sum_{i=1}^{N_t} \xi_i, \quad t \ge 0, \tag{3.1}$$

with $\mu \in \mathbb{R}, \sigma > 0$. Here $W = (W_t)_{t\ge 0}$ is a standard Brownian motion (or Wiener process) and $N = (N_t)_{t\ge 0}$ is an independent Poisson process with intensity $\lambda > 0$. The common distribution of the independent identically distributed (i.i.d) jumps ξ_i has a double exponential density

$$f_\xi(y) = p\gamma_1 e^{-\gamma_1 y}\mathbb{1}_{y>0} + q\gamma_2 e^{\gamma_2 y}\mathbb{1}_{y<0}, \tag{3.2}$$

with parameters $\gamma_1, \gamma_2, p, q > 0$ and $p + q = 1$. We call $\Theta := (\mu, \sigma, \lambda, p, \gamma_1, \gamma_2)$ the *characteristics* of X. It is known that the infinitesimal generator of X is

$$\begin{aligned} \mathcal{A}u(x) &= \mu u'(x) + \frac{1}{2}\sigma^2 u''(x) \\ &\quad + \lambda \int_{-\infty}^{\infty} [u(x+y) - u(y)] f_\xi(y) dy, \end{aligned} \tag{3.3}$$

and its Laplace exponent is given by

$$\begin{aligned} G(\delta) &= G(\delta; \Theta) \\ &= \mu\delta + \frac{1}{2}\sigma^2\delta^2 + \lambda\left(\frac{p\gamma_1}{\gamma_1 - \delta} + \frac{q\gamma_2}{\gamma_2 + \delta} - 1\right). \end{aligned} \tag{3.4}$$

[2] Noting that $X^{(+)}$ is nondecreasing and $X_t^{(+)} \ge 0$ for all $t \ge 0$, we have

$$Y_t - Y_t^{(-)} = X_t^{(+)} + \inf_{s\le t}(X_s^{(-)} \wedge 0) - \inf_{s\le t}[(X_s^{(-)} + X_s^{(+)}) \wedge 0] \ge 0,$$

which yields $\tau_x^{(-)} > \tau_x$.

[3] If τ_x is the first passage time of a reflected Brownian motion, we know from Proposition II.3.7 in Revuz and Yor (1999), p. 71 that

$$\mathbb{E}\left[e^{-\lambda\tau_x}\right] = \left(\cosh\left(x\sqrt{2\lambda}\right)\right)^{-1}, \qquad \mathbb{E}[\tau_x] = -\left.\frac{\partial\mathbb{E}\left[e^{-\lambda\tau_x}\right]}{\partial\lambda}\right|_{\lambda=0} = x^2.$$

[4] In fact the explicit solution is also available under a more general setting, i.e., the so-called Lévy phase-type assumption studied in Asmussen et al. (2004). However in order to obtain some simple results and avoid the direct usage of Wiener–Hopf factorization theory, we here only investigate the double exponential jump diffusion.

Recall that I and S are the running infimum and supremum of the surplus process X, and that $Y = X - I$ and $\hat{Y} = S - X$ are the surplus process X reflected at its running infimum I and at its running supremum S, respectively. We denote by $\hat{\Theta} := (-\mu, \sigma, \lambda, 1-p, \gamma_2, \gamma_1)$ the characteristic of the dual process $\hat{X} = -X$, and recall that the process $\hat{Y} = S - X = \hat{X} - \hat{I}$ is the dual process $\hat{X}$ reflected at its running infimum.

First, we have the following result concerning the solutions to the Cramér–Lundberg equation (see Lemma 2.1 in Kou and Wang (2003)).

Lemma 3.1. *For each $r > 0$, the Cramér–Lundberg equation*

$$G(\delta) = r, \tag{3.5}$$

has exactly four roots δ_i with $i = 1, 2, 3, 4$ such that

$$-\infty < \delta_1 < -\gamma_2 < \delta_2 < 0 < \delta_3 < \gamma_1 < \delta_4 < \infty.$$

In addition, define the overall drift of X by

$$\bar{\mu} := \mu + \frac{p\lambda}{\gamma_1} - \frac{q\lambda}{\gamma_2},$$

then, as $r \to 0$,

$$\delta_1 \to \delta_1^*, \qquad \delta_2 \to \begin{cases} 0 & \text{if } \bar{\mu} \le 0, \\ \delta_2^* & \text{if } \bar{\mu} > 0, \end{cases}$$

$$\delta_3 \to \begin{cases} 0 & \text{if } \bar{\mu} \ge 0, \\ \delta_3^* & \text{if } \bar{\mu} < 0, \end{cases} \quad \text{and} \quad \delta_4 \to \delta_4^*,$$

where $\delta_1^*, \delta_2^*, \delta_3^*$ and δ_4^* are defined as the unique roots such that

$$G(\delta_1^*) = G(\delta_2^*) = G(\delta_3^*) = G(\delta_4^*) = 0,$$
$$-\infty < \delta_1^* < -\gamma_2 < \delta_2^* < 0 < \delta_3^* < \gamma_1 < \delta_4^* < \infty.$$

We next prove a lemma similar to Lemma 2.1. We will find that, for the double exponential jump diffusion, a more general version holds. To this end, define

$$\varphi(x) = \mathbb{E}_x\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right], \tag{3.6}$$

for some nonnegative measurable function η on $[0, \infty)$. Then we have

Lemma 3.2. *Let $a > 0$ and $r > 0$. If $\eta(0) < \infty$ and $\int_0^\infty \eta(x)\gamma_1 e^{-\gamma_1 x}dx < \infty$, then $\varphi'(0+) = 0$.*

Proof. By a similar procedure as the proof of Lemma 2.1, we get, for $0 < x < a$

$$\begin{aligned}\mathbb{E}\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right] &\ge \mathbb{E}\left[e^{-r\tau_x}; Y_{\tau_x} = x\right]\mathbb{E}_x\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right] \\ &\ge \mathbb{E}\big[e^{-r\tau_x^{(-)}}; N^{(+)}_{\tau_x^{(-)}} = 0\big]\mathbb{E}_x\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right] \\ &= \mathbb{E}\big[e^{-(r+p)\tau_x^{(-)}}\big]\mathbb{E}_x\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right],\end{aligned}$$

from which we can get that $\varphi'(0+) \le 0$. On the other hand, we can prove that, conditional on $Y_{\tau_a} - a > 0$, the overshoot $Y_{\tau_a} - a$ is independent of τ_a and has the exponential distribution with parameter γ_1 (see also Proposition 2.1 in Kou and Wang (2003)). We thus have

$$\begin{aligned}\mathbb{E}\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right] \le \mathbb{E}&\left[e^{-r\tau_x}; Y_{\tau_x} = x\right] \\ &\times \mathbb{E}_x\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right] \\ &+ C\mathbb{P}[Y_{\tau_x} > x],\end{aligned}$$

with $C = \max\{\eta(0), \int_0^\infty \eta(x)\gamma_1 e^{-\gamma_1 x}dx\} < \infty$, and

$$\begin{aligned}\varphi'(0+) &= \lim_{x\downarrow 0}\frac{\mathbb{E}_x\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right] - \mathbb{E}\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right]}{x} \\ &\ge \lim_{x\downarrow 0}\frac{\mathbb{E}_x\left[e^{-r\tau_a}\eta(Y_{\tau_a} - a)\right]\left(1 - \mathbb{P}\left[Y_{\tau_x} = x\right]\right) - C(1 - \mathbb{P}[Y_{\tau_x} = x])}{x}.\end{aligned}$$

Note that

$$0 \le \lim_{x\downarrow 0}\frac{1 - \mathbb{P}[Y_{\tau_x} = x]}{x} \le \lim_{x\downarrow 0}\frac{1 - \mathbb{E}[e^{-p\tau_x^{(-)}}]}{x} = 0.$$

It follows that $\varphi'(0+) \ge 0$, which concludes the proof. □

From Lemmas 2.1 and 3.2, we find that under the double exponential assumption almost all of the most important functions in insurance satisfy the smooth pasting property, which is a homogeneous Neumann boundary condition at the reflecting barrier 0. We next find the infinitesimal generator of the reflected Lévy process Y.

Proposition 3.1. *Let X be the double exponential jump diffusion given by (3.1). The infinitesimal generator of the reflected Lévy process $Y := X - I$ is given by*

$$\begin{aligned}\tilde{\mathcal{A}}u(x) = {}& \mu u'(x) + \frac{1}{2}\sigma^2 u''(x) \\ &+ \lambda\int_{-\infty}^{\infty}[u((x+y)\vee 0) - u(x)]f_\xi(y)dy,\end{aligned} \tag{3.7}$$

with the domain of definition

$$\mathcal{D}(\tilde{\mathcal{A}}) = \{u \in C^2[0,\infty) : u'(0) = 0\}. \tag{3.8}$$

Proof. Note that $Y = X - I$ and[5]

$$I_t^c = \int_{[0,t]}\mathbb{1}_{\{Y_u=0\}}dI_u^c = \int_{[0,t]}\mathbb{1}_{\{Y_{u-}=0\}}dI_u^c, \tag{3.10}$$

where $I_t^c = I_t - \sum_{s\le t}\Delta I_s$ is the continuous part of I with $\Delta I_t = I_t - I_{t-}$ being the jump at time t. Applying Itô's formula to $u(Y_t)$ for any function $u \in \mathcal{D}(\tilde{\mathcal{A}})$, we have

$$\begin{aligned}\mathbb{E}_x[u(Y_t)] = {}& u(x) + \mathbb{E}_x\left[\int_0^t \mu u'(Y_s)ds - \int_0^t u'(Y_{s-})dI_s^c\right. \\ &\left.+ \frac{1}{2}\int_0^t \sigma^2 u''(Y_s)ds\right] \\ &+ \lambda\int_0^t\int_{-\infty}^{\infty}(u((x+y)\vee 0) \\ &- u(x))f_\xi(y)dyds \qquad (3.11) \\ = {}& u(x) + \mathbb{E}_x\left[\int_0^t \mu u'(Y_s)ds + \frac{1}{2}\int_0^t \sigma^2 u''(Y_s)ds\right] \\ &+ \lambda\int_0^t\int_{-\infty}^{\infty}(u((x+y)\vee 0) \\ &- u(x))f_\xi(y)dyds. \qquad (3.12)\end{aligned}$$

By the definition of infinitesimal generator, we have

$$\begin{aligned}\tilde{\mathcal{A}}u(x) &:= \lim_{t\downarrow 0}\frac{\mathbb{E}_x[u(Y_t)] - u(x)}{t} \qquad (3.13) \\ &= \mu u'(x) + \frac{1}{2}\sigma^2 u''(x) + \lambda\int_{-\infty}^{\infty}[u((x+y)\vee 0) \\ &\quad - u(x)]f_\xi(y)dy. \qquad (3.14)\end{aligned}$$

This completes the proof. □

[5] It is worth noting that (see, e.g., Cooper et al. (2001) and Andersen and Asmussen (2009))

$$I_t = \int_{[0,t]}\mathbb{1}_{\{Y_u=0\}}dI_u \neq \int_{[0,t]}\mathbb{1}_{\{Y_{u-}=0\}}dI_u, \tag{3.9}$$

since on $\{\Delta I_t = I_t - I_{t-} > 0\}$, we have $0 = Y_t < Y_{t-}$ almost surely.

Now we are in a position to present the Laplace transforms related to the entrance times and the overshoots of the reflected double exponential jump diffusion model.

Proposition 3.2. *Let $\delta_1 < -\gamma_2 < \delta_2 < 0 < \delta_3 < \gamma_1 < \delta_4$ be the four roots to the Eq. (3.5). Then we have, for $0 \le x < a$ and $y \ge 0$*

$$\mathbb{E}_x\left[e^{-r\tau_a}\right] = \sum_{i=1}^{4} C_i e^{-\delta_i x}, \tag{3.15}$$

$$\mathbb{E}_x\left[e^{-r\tau_a} 1_{\{Y_{\tau_a}-a>y\}}\right] = e^{-\gamma_1 y}\sum_{i=1}^{4} \bar{C}_i e^{-\delta_i x}, \tag{3.16}$$

where C_i's satisfy the following system of linear equations

$$\sum_{i=1}^{4} C_i \frac{\gamma_1}{\delta_i+\gamma_1} e^{-\delta_i a} = 1, \tag{3.17a}$$

$$\sum_{i=1}^{4} C_i \frac{\delta_i}{\delta_i-\gamma_2} = 0, \tag{3.17b}$$

$$\sum_{i=1}^{4} C_i e^{-\delta_i a} = 1, \tag{3.17c}$$

$$\sum_{i=1}^{4} C_i \delta_i = 0, \tag{3.17d}$$

and $\bar{C}_i$'s satisfy the following system of linear equations

$$\sum_{i=1}^{4} \bar{C}_i \frac{\gamma_1}{\delta_i+\gamma_1} e^{-\delta_i a} = 1, \tag{3.18a}$$

$$\sum_{i=1}^{4} \bar{C}_i \frac{\delta_i}{\delta_i-\gamma_2} = 0, \tag{3.18b}$$

$$\sum_{i=1}^{4} \bar{C}_i e^{-\delta_i a} = 0, \tag{3.18c}$$

$$\sum_{i=1}^{4} \bar{C}_i \delta_i = 0. \tag{3.18d}$$

Proof. See Appendix. □

Remark 3.1. By a similar argument as the one used in the proof of Proposition 3.2, we can derive the formulas for u_u and u_d in Corollary 2.1, and then verify that the formula (3.15) is consistent with (2.28).

In Proposition 3.2, the conditions (3.17c) and (3.18c) are the so-called continuous pasting conditions such that those functions are continuous at point a. The conditions (3.17d) and (3.18d) are the smooth pasting conditions obtained in Lemmas 2.1 and 3.2. Since we are considering the double exponential jump diffusion, both X and its dual process $\hat{X} = -X$ satisfy the assumptions in Theorem 2.1, and Lemmas 2.1 and 3.2. This results in the following proposition related to the process $\hat{Y} = \hat{X} - \hat{I}$.

Proposition 3.3. *Let $\hat{\delta}_1 < -\gamma_1 < \hat{\delta}_2 < 0 < \hat{\delta}_3 < \gamma_2 < \hat{\delta}_4$ be the four roots to the equation $\hat{G}(\hat{\delta}) = G(\hat{\delta}; \hat{\Theta}) = r$. Then we have, for $0 \le x < a$*

$$\mathbb{E}_{-x}\left[e^{-r\hat{\tau}_a}\right] = \sum_{i=1}^{4} \hat{C}_i e^{-\hat{\delta}_i x}, \tag{3.19}$$

$$\mathbb{E}_{-x}\left[e^{-r\hat{\tau}_a} 1_{\{\hat{Y}_{\hat{\tau}_a}-a>y\}}\right] = e^{-\gamma_2 y}\sum_{i=1}^{4} \hat{\hat{C}}_i e^{-\hat{\delta}_i x}, \tag{3.20}$$

where $\hat{C}_i$'s satisfy the following system of linear equations

$$\sum_{i=1}^{4} \hat{C}_i \frac{\gamma_2}{\hat{\delta}_i+\gamma_2} e^{-\hat{\delta}_i a} = 1, \qquad \sum_{i=1}^{4} \hat{C}_i \frac{\hat{\delta}_i}{\hat{\delta}_i-\gamma_1} = 0, \tag{3.21a}$$

$$\sum_{i=1}^{4} \hat{C}_i e^{-\hat{\delta}_i a} = 1, \qquad \sum_{i=1}^{4} \hat{C}_i \hat{\delta}_i = 0, \tag{3.21b}$$

and $\hat{\hat{C}}_i$'s satisfy the system of linear equations

$$\sum_{i=1}^{4} \hat{\hat{C}}_i \frac{\gamma_2}{\hat{\delta}_i+\gamma_2} e^{-\hat{\delta}_i a} = 1, \qquad \sum_{i=1}^{4} \hat{\hat{C}}_i \frac{\hat{\delta}_i}{\hat{\delta}_i-\gamma_1} = 0, \tag{3.22a}$$

$$\sum_{i=1}^{4} \hat{\hat{C}}_i e^{-\hat{\delta}_i a} = 0, \qquad \sum_{i=1}^{4} \hat{\hat{C}}_i \hat{\delta}_i = 0. \tag{3.22b}$$

Proof. The result is a direct consequence of Proposition 3.2 by noting that, under $\mathbb{P}_{-x}$, the process $\hat{Y}$ starts at x and has the same distribution as the dual process $\hat{X}$ (with characteristic $\hat{\Theta}$) reflected at its running infimum. □

Remark 3.2. By symmetry, it is easy to see that $\hat{\delta}_i = -\delta_{5-i}$, $i = 1, 2, 3, 4$.

3.2. Laplace transform of ruin time, deficit at ruin and expected discounted dividends

The following theorem is a direct consequence of Proposition 2.1(I) and Proposition 3.3.

Theorem 3.1. *Suppose $\hat{\delta}_i, \hat{C}_i$, $i = 1, 2, 3, 4$, are given in Proposition 3.3 with a replaced by b therein. The Laplace transform of the ruin time can be expressed as*

$$\mathbb{E}_x\left[e^{-r\tilde{\tau}_b}\right] = \sum_{i=1}^{4} \hat{C}_i e^{-\hat{\delta}_i (b-x)}. \tag{3.23}$$

The next theorem presents the distribution and the mean of the deficit at ruin $|U^b_{\tilde{\tau}_b}| = -U^b_{\tilde{\tau}_b}$.

Theorem 3.2. *Suppose $\hat{\delta}_i, \hat{C}_i, \hat{\hat{C}}_i$, $i = 1, 2, 3, 4$, are given in Proposition 3.3 with a replaced by b therein. Then, for $y \ge 0$,*

$$\mathbb{E}_x\left[e^{-r\tilde{\tau}_b} \mathbb{1}_{\{|U^b_{\tilde{\tau}_b}|>y\}}\right] = e^{-\gamma_2 y}\sum_{i=1}^{4} \hat{\hat{C}}_i e^{-\hat{\delta}_i (b-x)}, \tag{3.24a}$$

$$\mathbb{E}_x\left[e^{-r\tilde{\tau}_b} \mathbb{1}_{\{|U^b_{\tilde{\tau}_b}|=0\}}\right] = \sum_{i=1}^{4} (\hat{C}_i - \hat{\hat{C}}_i) e^{-\hat{\delta}_i (b-x)}. \tag{3.24b}$$

Moreover the expected discounted deficit at ruin is given by

$$\mathbb{E}_x\left[e^{-r\tilde{\tau}_b} \left|U^b_{\tilde{\tau}_b}\right|\right] = \frac{1}{\gamma_2}\sum_{i=1}^{4} \hat{\hat{C}}_i e^{-\hat{\delta}_i (b-x)}. \tag{3.25}$$

Proof. The first two equations are direct consequences of Proposition 2.1(III) and Proposition 3.3. The last conclusion follows by noting that

$$\mathbb{E}_x\left[e^{-r\tilde{\tau}_b} \left|U^b_{\tilde{\tau}_b}\right|\right] = \int_0^{+\infty} y \left(-\frac{\partial}{\partial y}\mathbb{E}_x\left[e^{-r\tilde{\tau}_b} \mathbb{1}_{\{|U^b_{\tilde{\tau}_b}|>y\}}\right]\right) dy.$$

The proof is complete. □

Finally, we compute the expected total discounted dividends up to ruin. Recall from (2.2) that T_a^+ and T_a^- are the entrance times of the initial surplus process X into $(a, +\infty)$ and $(-\infty, -a)$, respectively. Theorem 3.1 in Kou and Wang (2003) obtained explicit expressions for the quantities

$$\mathbb{E}[\exp(-rT_a^+)], \qquad \mathbb{E}[\exp(-rT_a^+)\mathbb{1}_{\{X_{T_a^+}=a\}}],$$
$$\mathbb{E}[\exp(-rT_a^+)\mathbb{1}_{\{X_{T_a^+}-a>y\}}] \quad \text{for } y \geq 0.$$

Now we can present an explicit expression for the total expected discounted dividends.

Theorem 3.3. *Let $b > 0$. For $x \in [0, b]$, the total expected discounted dividends can be expressed as*

$$D(b; x) := \mathbb{E}_x\left[\int_0^{\tilde{\tau}_b} e^{-rt}dL_t^b\right] = \frac{1}{\gamma_1(\delta_4-\delta_3)}(k(x-b)-h(x,b)), \tag{3.26}$$

where

$$k(x-b) = \frac{(\gamma_1-\delta_3)\delta_4}{\delta_3}e^{(x-b)\delta_3} + \frac{(\delta_4-\gamma_1)\delta_3}{\delta_4}e^{(x-b)\delta_4}, \tag{3.27a}$$

$$h(x,b) = \left(\frac{(\gamma_1-\delta_3)\delta_4}{\delta_3}e^{-b\delta_3} + \frac{(\delta_4-\gamma_1)\delta_3}{\delta_4}e^{-b\delta_4}\right) \times \sum_{i=1}^4 \hat{C}_i e^{\hat{\delta}_i(x-b)} - \left(\frac{\delta_4(\gamma_1-\delta_3)}{\delta_3+\gamma_2}e^{-b\delta_3} + \frac{\delta_3(\delta_4-\gamma_1)}{\delta_4+\gamma_2}e^{-b\delta_4}\right) \times \sum_{i=1}^4 \hat{\hat{C}}_i e^{\hat{\delta}_i(x-b)}, \tag{3.27b}$$

with $0 < \delta_3 < \gamma_1 < \delta_4$ being the only two positive roots of the equation $G(\delta) = r$, $\hat{\delta}_1 < -\gamma_1 < \hat{\delta}_2 < 0 < \hat{\delta}_3 < \gamma_2 < \hat{\delta}_4$ being the four roots to the equation $\hat{G}(\hat{\delta}) = r$, and $\hat{C}_i, \hat{\hat{C}}_i$, $i = 1, 2, 3, 4$, given by Proposition 3.3 with a replaced by b. Moreover, if $x > b$, we have

$$D(b; x) = x - b + D(b; b). \tag{3.28}$$

Proof. From Theorem 2.1, we have

$$\mathbb{E}_x\left[\int_0^{\tilde{\tau}_b} e^{-rt}dL_t^b\right] = k(x-b) - \mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b}k(-\hat{Y}_{\hat{\tau}_b})\right],$$

where, for $x \geq 0$,

$$k(-x) = \mathbb{E}[e^{-rT_x^+}(X_{T_x^+}-x)] + \mathbb{E}[e^{-rT_x^+}]\int_0^\infty \mathbb{E}[e^{-rT_z^+}]dz.$$

Note that $\mathbb{E}[e^{-rT_x^+}]$ and $\mathbb{E}[e^{-rT_x^+}\mathbb{1}_{\{X_{T_x^+}-x>y\}}]$ for $y \geq 0$ are known (see (3.1) and (3.2) in Kou and Wang (2003)). We have

$$\int_0^\infty \mathbb{E}[e^{-rT_z^+}]dz = \frac{1}{\delta_3} + \frac{1}{\delta_4} - \frac{1}{\gamma_1},$$

and

$$\mathbb{E}\left[e^{-rT_x^+}(X_{T_x^+}-x)\right] = \int_0^\infty y\left(-\frac{\partial}{\partial y}\mathbb{E}\left[\exp(-rT_x^+)\mathbb{1}_{\{X_{T_x^+}-x>y\}}\right]\right)dy = \frac{(\delta_4-\gamma_1)(\gamma_1-\delta_3)}{\gamma_1^2(\delta_4-\delta_3)}\left(e^{-x\delta_3} - e^{-x\delta_4}\right).$$

The expression for $k(\cdot)$ follows after some simple algebra. On the other hand, from (3.19) to (3.20), we have

$$\mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b}\mathbb{1}_{\{\hat{Y}_{\hat{\tau}_b}-b>y\}}\right] = e^{-\gamma_2 y}\sum_{i=1}^4 \hat{\hat{C}}_i e^{\hat{\delta}_i(x-b)}, \tag{3.29}$$

$$\mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b}\mathbb{1}_{\{\hat{Y}_{\hat{\tau}_b}=b\}}\right] = \sum_{i=1}^4 (\hat{C}_i - \hat{\hat{C}}_i)e^{\hat{\delta}_i(x-b)}. \tag{3.30}$$

It follows that

$$\begin{aligned}\mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b}k(-\hat{Y}_{\hat{\tau}_b})\right] &= \int_0^\infty k(-(b+y))\left(-\frac{\partial}{\partial y}\mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b}\mathbb{1}_{\{\hat{Y}_{\hat{\tau}_b}-b>y\}}\right]\right)dy \\ &\quad + k(-b)\mathbb{E}_{x-b}\left[e^{-r\hat{\tau}_b}\mathbb{1}_{\{\hat{Y}_{\hat{\tau}_b}=b\}}\right] \\ &= \sum_{i=1}^4\left[\hat{\hat{C}}_i\int_0^\infty k(-(b+y))\gamma_2 e^{-\gamma_2 y}dy + k(-b)(\hat{C}_i - \hat{\hat{C}}_i)\right]e^{\hat{\delta}_i(x-b)}.\end{aligned}$$

The proof is completed by substituting (3.27a) into the above equation and using some algebra. □

Remark 3.3. Note that $\sum_{i=1}^4 \hat{C}_i e^{-\hat{\delta}_i b} = 1$ and $\sum_{i=1}^4 \hat{\hat{C}}_i e^{-\hat{\delta}_i b} = 0$. It is easy to see from (3.26) that $D(b; 0) = 0$ for all $b \geq 0$. This is consistent with intuition since $\tilde{\tau}_b = 0$ a.s. when $\sigma > 0$.

4. Numerical results: the optimal barrier strategy

In this section, we present some numerical results concerning the optimal dividend barrier for the double exponential jump diffusion model. The program is coded in Mathematica®. The underlying surplus process is described by a double exponential jump diffusion with parameters

$$\Theta = (\mu, \sigma, \lambda, p, \gamma_1, \gamma_2) = (0.1, 0.2, 20, 0.8, 60, 20). \tag{4.1}$$

So the mean return per unit time is $\bar{\mu} = \mu + \frac{p\lambda}{\gamma_1} - \frac{(1-p)\lambda}{\gamma_2} = \frac{1}{6}$. We can use some Laplace inversion algorithm (e.g., the Gaver–Stehfest algorithm used in Kou and Wang (2003) and the Gaver–Wynn–Rho algorithm used in Bo et al. (2011b,a); see also Abate and Whitt (1992) and Abate and Valkó (2004)) to invert the Laplace transform of the ruin time to get its distribution. The distribution and the mean of the deficit at ruin can be obtained from Theorem 3.2 by letting $r \downarrow 0$. In this section we will concentrate on finding the optimal dividend barrier. Recall from Theorem 3.3 that

$$D(b; x) = \begin{cases}\frac{1}{\gamma_1(\delta_4-\delta_3)}(k(x-b)-h(x,b)), & 0 \leq x \leq b, \\ x - b + D(b; b), & x > b,\end{cases} \tag{4.2}$$

where the functions k and h are given in (3.27). Denote

$$\hat{b} = \arg\max_{b\geq 0} D(b; b) - b. \tag{4.3}$$

We first study the optimal barrier b^* for different values of the initial surplus x. The left panel of Fig. 1 shows the function $D(b; x)$ against the dividend barrier b. The optimal barrier parameter b^* and the optimal value $D(b^*; x)$ are reported in Table 1. The left panel of Fig. 2 shows the optimal barrier b^* versus the initial surplus x and right panel depicts the optimal value $D(b^*; x)$ versus the initial surplus x. We find that when the initial surplus x is smaller than $\hat{b}$, the optimal dividend barrier b^* is smaller than $\hat{b}$

Table 1
The optimal dividend barrier b^* versus the initial surplus x. The model parameters are given by (4.1) and the discount rate $r = 0.02$.

x	10^{-10}	10^{-3}	1/10	1/2	3/2	5/2	3.0355	7/2	9/2	11/2
b^*	0.6315	0.6320	0.6903	0.9353	1.7044	2.5693	3.0355	3.0355	3.0355	3.0355
$D(b^*, x)$	10^{-9}	0.0062	0.2434	0.9337	2.4570	3.6990	4.2649	4.7293	5.7293	6.7293

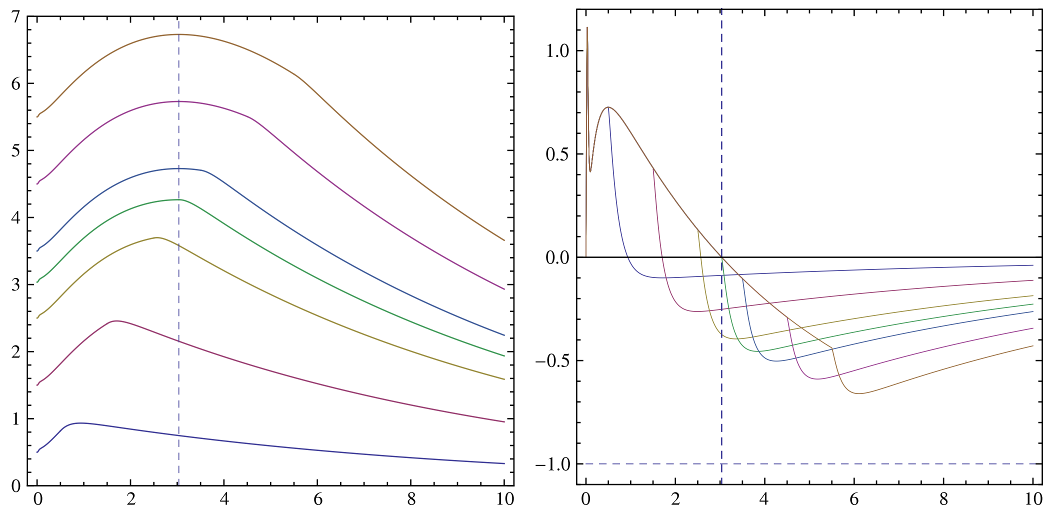

Fig. 1. *Left:* expected discounted dividends $D(b; x)$ versus barrier parameter b with initial surplus $x = 1/2, 3/2, 5/2, 3.0355, 7/2, 9/2, 11/2$ (from the bottom up). Right: the derivative (with respect to b) of $D(b; x)$ versus barrier parameter b with initial surplus $x = 1/2, 3/2, 5/2, 3.0355, 7/2, 9/2, 11/2$ (from the top down as $b \in [6, 8]$). The optimal barrier parameter b^* and the optimal value $D(b^*; x)$ are reported in Table 1. The vertical dashed line corresponds to the value $\hat{b}$ defined in (4.3). In both of the two plots the discount rate is $r = 0.02$.

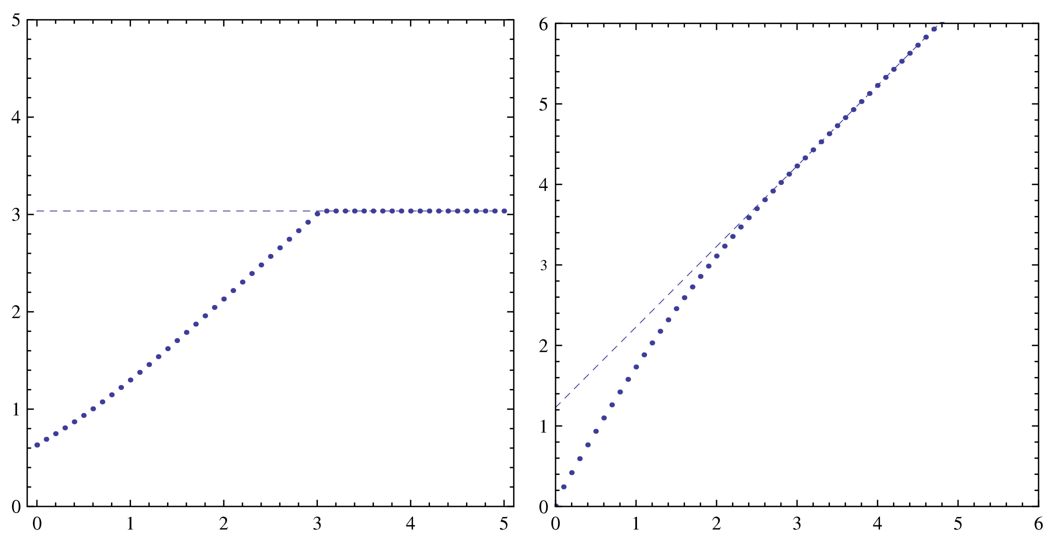

Fig. 2. Left: optimal dividend barrier b^* versus initial surplus x. The horizontal dashed line is at the level 3.0355. Right: optimal dividend value $D(b^*; x)$ versus initial surplus x. The dashed line has slope 1. In both of the two plots the discount rate is $r = 0.02$. Some of the values of b^* and $D(b^*; x)$ are reported in Table 1.

as well and increases as the initial surplus x increases.[6] While if the initial surplus $x \geq \hat{b}$, the optimal dividend barrier will be determined by $b^* = \hat{b}$, which is independent of x. In both cases the optimal dividend value is a increasing function of the initial surplus (note that when $x \geq \hat{b}$ the optimal dividend value $D(b^*; x)$ is a linear function of x with slope 1). We also find that, as the initial surplus tends to zero, the optimal barrier converges to a positive value, while the optimal dividend value goes to zero (because there is a diffusion part). It is worth noting that for the spectrally negative risk model the optimal dividend barrier is independent of the initial surplus x (see, e.g., Gerber (1969); Bühlmann (1970), (7.10) in Gerber and Shiu (1998), (7.3) in Gerber and Shiu (2004), Theorem 3 in Li and Wu (2008) and Theorem 1.2 in Kyprianou et al. (2010)).

We next study the optimal dividend barrier b^* versus the discount rate r. We find from Table 2 that the optimal dividend barrier b^* decreases as the discount rate r increases, which is consistent with intuition since the shareholders prefer earlier dividends when r is large and the dividends come earlier if the dividend barrier is smaller. Moreover, the optimal dividend barrier b^* (resp. the optimal expected discounted dividend $D(b^*; x)$) decreases to 0 (resp. x) as r increases to ∞.

Finally we take a look at the derivative of $D(b; x)$ with respect to b for some fixed $x > 0$. From Fig. 3 (see also the right panel of

[6] By applying the optimal dividend barrier of the initial surplus, one never gets up to those other (higher) barrier values that would be optimal for the higher surplus, which is a drawback of surplus-dependent 'optimal' barriers.

Table 2
The optimal dividend barrier b^* versus the discount rate r. The model parameters are given by (4.1) and the initial surplus $x = 1$.

r	100	1/2	1/5	1/10	1/20	1/100	10^{-3}	10^{-4}	10^{-5}	10^{-10}
b^*	0.0000	0.1398	0.3691	0.6950	1.0810	1.4619	1.9870	2.5012	3.0133	3.2140
$D(b^*; x)$	1.0000	1.0489	1.1034	1.2180	1.4482	1.8726	2.0441	2.0683	2.0714	2.0718

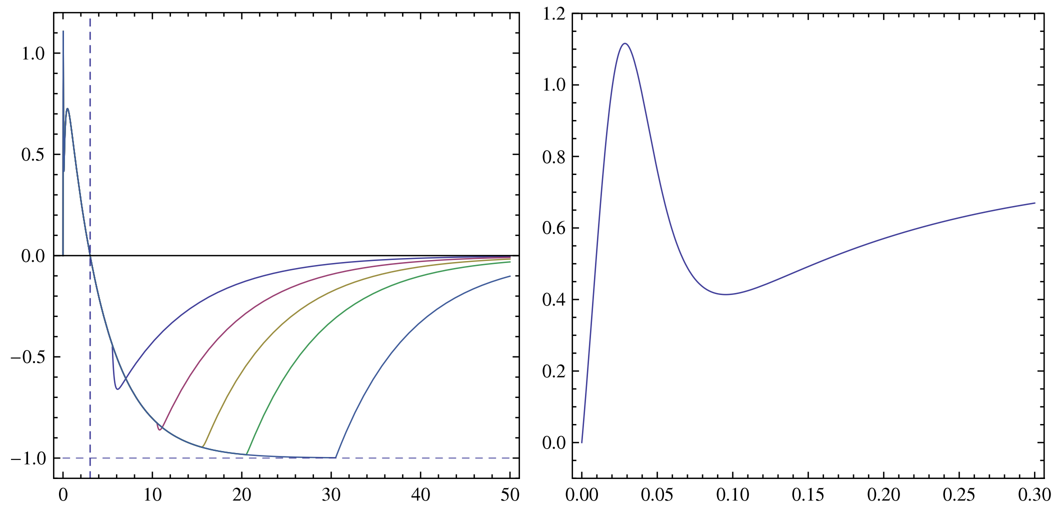

Fig. 3. LEFT: the derivative (with respect to b) of $D(b; x)$ versus barrier parameter b with initial surplus $x = 5, 10, 15, 20, 30$ (from the top down on $b > 20$). RIGHT: the local display of the plot in the left panel. In both of the two plots the discount rate is $r = 0.02$.

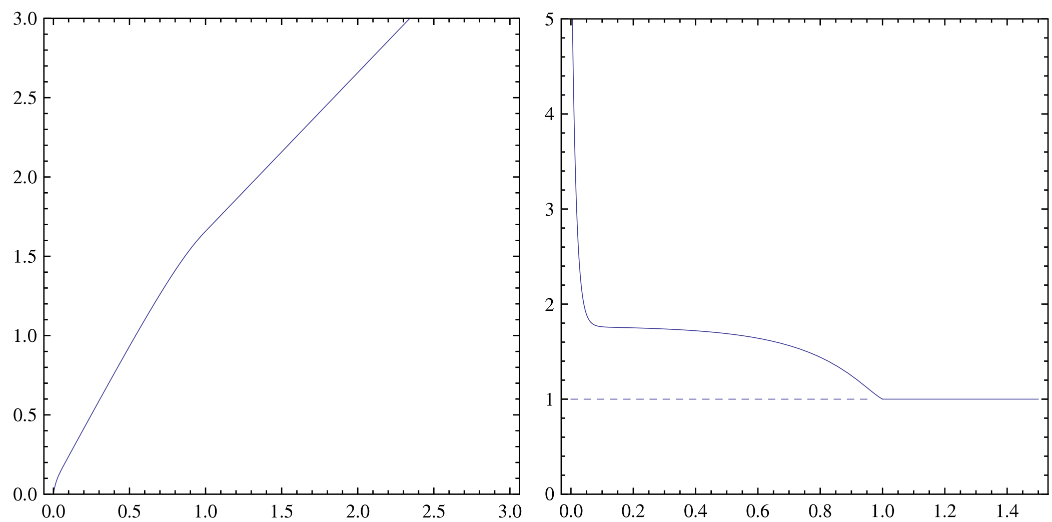

Fig. 4. LEFT: $D(b; x)$ versus initial surplus x with barrier parameter $b = 1$. RIGHT: the derivative (with respect to x) of $D(b; x)$ in the left panel versus initial surplus x. In both of the two plots the discount rate is $r = 0.02$.

Fig. 1) we find that: (1) $\frac{\partial D(b;x)}{\partial b} \geq -1$ and $\frac{\partial D(b;x)}{\partial b}|_{b=0} = 0$; (2) $\frac{\partial D(b;x)}{\partial b}$ is continuous on $[0, \infty)$; (3) for any $x > 0$, the optimal barrier b^* is the unique $b > 0$ such that $\frac{\partial D(b;x)}{\partial b} = 0$ (if $x > \hat{b}$, $b^* = \hat{b}$).

Moreover the numerical experiments also reveal that (see Fig. 4), for a fixed $b > 0$, the derivative of $D(b; x)$ with respect to x is a strictly positive continuous non-increasing function of x. Specifically, it holds that $\frac{\partial D(b;x)}{\partial x}|_{x=b-} = \frac{\partial D(b;x)}{\partial x}|_{x=b+} = \frac{\partial D(b;x)}{\partial x}|_{x=z} = 1$, for all $z > b$.

However, due to the complexity of the expression of $D(b; x)$ (since $\hat{C}_i$'s and $\hat{\hat{C}}_i$'s are solutions to some systems of linear equations), it seems difficult to prove all of the above observations rigorously. We leave the related issues as open problems for further research, which may be hard to solve (if it can be), since the two-sided jumps always result in a complex structure for the expected discounted dividends function $D(b; x)$.

5. Conclusion

This paper studied a general Lévy risk model with two-sided jumps and a constant dividend barrier. For a general Lévy risk process, we expressed the Laplace transform of the ruin time, the deficit at ruin and the expected discounted dividends in terms of the first passage problem of the Lévy process reflected at its running maximum (or the dual Lévy process reflected at its infimum). For the Lévy risk model which can be expressed as the sum of a spectrally negative Lévy process with unbounded variation and a subordinator, we showed that the smooth pasting condition holds for the Laplace transform of the upward entrance time of the Lévy process reflected at its running infimum. A more general smooth pasting condition was proved for the double exponential risk model. Based on those results, we present explicit expressions for the Laplace transform of the ruin time, the deficit at ruin and the expected discounted dividends when the underlying risk model is a double exponential jump diffusion. All these expressions were explicit functions of the initial surplus and the dividend barrier, and could be easily implemented to get numerical results. Numerical experiments concerning the optimal dividend barrier were also presented. The most important empirical finding is that the optimal dividend barrier would depend on the initial surplus if the initial surplus is less than some critical value. We also find that the optimal barrier converges to a positive value as the initial surplus decreases to zero. On the other hand, if the initial surplus is greater than or equal to the critical value, the optimal dividend barrier will equal the critical value. The dependence on the initial surplus of the optimal dividend barrier stems from the incorporation of two-sided jumps in the risk model. However it seems to be hard to verify these empirical findings rigorously, which is due to the complicated structure of the dividend as a function of the barrier parameter. We leave it as an open problem for further research.

Almost all our results related to the double exponential jump diffusion model can be generalized to a more general class of Lévy process, the so-called Lévy phase-type model (which is known to be dense in the class of all Lévy processes), with some effort (see Asmussen et al. (2004) for related results on Lévy phase-type model). Moreover, under our settings, the ruin-free risk model considered in Section 4 of Avram et al. (2007) can also be studied.

Another interesting problem is to provide necessary and sufficient conditions (in terms of the Lévy characteristics) under which the lower barrier 0 of the reflected Lévy process is an instantaneous reflecting barrier (or the Laplace transform of upward entrance time exhibit smooth pasting at 0), so that its infinitesimal generator should be defined on a functional space with homogeneous Neumann boundary condition at 0. For smooth pasting in optimal stopping problems, the interested reader is referred to Alili and Kyprianou (2005).

Acknowledgments

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Appendix. Proofs

Proof of Proposition 3.2. Recall the infinitesimal generator of the reflected process Y

$$\tilde{\mathcal{A}}u(x) = \mu u'(x) + \frac{1}{2}\sigma^2 u''(x) + \lambda \int_{-\infty}^{\infty} [u((x+y)\vee 0) - u(x)] f_\xi(y)dy, \tag{A.1}$$

with the domain of definition $\mathcal{D}(\tilde{\mathcal{A}}) = \{u \in C^2[0,\infty) : u'(0) = 0\}$. Define function u by (see (3.15))

$$u(x) := \begin{cases} \sum_{i=1}^4 C_i e^{-\delta_i x}, & 0 \le x < a, \\ 1, & x \ge a, \end{cases} \tag{A.2}$$

where C_i's satisfy the system of linear equations (3.17). From (3.17c) we have u is continuous at a. By (3.17d), it follows that $u \in \mathcal{D}(\tilde{\mathcal{A}})$. Moreover, by doing the integration in three regions $(-\infty, -x]$, $(-x, a-x)$ and $[a-x, \infty)$, we get

$$\begin{aligned}\tilde{\mathcal{A}}u(x) - ru(x) &= \sum_{i=1}^4 C_i e^{-\delta_i x}\phi(\delta_i) + \lambda q e^{-\gamma_2 x}\sum_{i=1}^4 C_i \frac{\delta_i}{\delta_i - \gamma_2} \\ &\quad - \lambda p e^{-\gamma_1(a-x)}\left(\sum_{i=1}^4 C_i \frac{\gamma_1}{\delta_i+\gamma_1} e^{-\delta_i a} - 1\right),\end{aligned} \tag{A.3}$$

where $\phi(\delta) = G(\delta) - r$. Recall from (3.17) that

$$\sum_{i=1}^4 C_i \frac{\gamma_1}{\delta_i+\gamma_1} e^{-\delta_i a} = 1, \qquad \sum_{i=1}^4 C_i \frac{\delta_i}{\delta_i - \gamma_2} = 0.$$

We have

$$\tilde{\mathcal{A}}u(x) - ru(x) = 0, \quad 0 \le x < a. \tag{A.4}$$

Now by Itô's formula for jump processes (see, e.g., Theorem 32 in Protter (2004)) and the boundedness of the function u, we have

$$\{M_t := e^{-r(t\wedge\tau_a)} u(Y_{t\wedge\tau_a}),\ t \ge 0\} \tag{A.5}$$

is a martingale. By the martingale property

$$u(x) = \mathbb{E}_x\left[e^{-r(t\wedge\tau_a)} u(Y_{t\wedge\tau_a})\right] \quad \text{for each } t \ge 0. \tag{A.6}$$

Note that $u(Y_{\tau_a}) = 1$. By the Lebesgue dominated convergence theorem, we have

$$\begin{aligned}u(x) &= \lim_{t\to\infty} \mathbb{E}_x\left[e^{-r(t\wedge\tau_a)} u(Y_{t\wedge\tau_a})\right] = \mathbb{E}_x\left[e^{-r\tau_a} u(Y_{\tau_a})\right] \\ &= \mathbb{E}_x\left[e^{-r\tau_a}\right].\end{aligned} \tag{A.7}$$

We have proved (3.15).

We next prove (3.16). Define

$$u(x) := \begin{cases} e^{-\gamma_1 y}\sum_{i=1}^4 \bar{C}_i e^{-\delta_i x}, & 0 \le x < a, \\ 0, & a \le x \le a+y, \\ 1, & x > a+y, \end{cases} \tag{A.8}$$

where $\bar{C}_i$'s satisfy the system of linear equations (3.18). From (3.18c) we have u is continuous at a. By (3.18d), it follows that $u \in \mathcal{D}(\tilde{\mathcal{A}})$. Moreover, by doing the integration in three regions $(-\infty, -x]$, $(-x, a-x)$ and $[a+y-x, \infty)$, we get

$$\begin{aligned}\tilde{\mathcal{A}}u(x) - ru(x) &= e^{-\gamma_1 y}\sum_{i=1}^4 \bar{C}_i e^{-\delta_i x}\phi(\delta_i) \\ &\quad + \lambda q e^{-\gamma_2 x - \gamma_1 y}\sum_{i=1}^4 \bar{C}_i \frac{\delta_i}{\delta_i - \gamma_2} \\ &\quad - \lambda p e^{-\gamma_1(a+y-x)} \\ &\quad \times \left(\sum_{i=1}^4 \bar{C}_i \frac{\gamma_1}{\delta_i+\gamma_1} e^{-\delta_i a} - 1\right),\end{aligned} \tag{A.9}$$

where $\phi(\delta) = G(\delta) - r$. Recall from (3.18) that

$$\sum_{i=1}^4 \bar{C}_i \frac{\gamma_1}{\delta_i+\gamma_1} e^{-\delta_i a} = 1, \qquad \sum_{i=1}^4 \bar{C}_i \frac{\delta_i}{\delta_i - \gamma_2} = 0.$$

We have

$$\tilde{\mathcal{A}}u(x) - ru(x) = 0, \quad 0 \le x < a. \tag{A.10}$$

Now by a similar martingale argument, we have

$$\begin{aligned}u(x) &= \lim_{t\to\infty} \mathbb{E}_x\left[e^{-r(t\wedge\tau_a)} u(Y_{t\wedge\tau_a})\right] = \mathbb{E}_x\left[e^{-r\tau_a} u(Y_{\tau_a})\right] \\ &= \mathbb{E}_x\left[e^{-r\tau_a}\mathbb{1}_{\{Y_{\tau_a} - a > y\}}\right],\end{aligned} \tag{A.11}$$

since $u(Y_{\tau_a}) = \mathbb{1}_{\{Y_{\tau_a} - a > y\}}$. We have proved (3.16). □

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