

Math 347F1, HW #8

Due Friday, April 3, 2009

1. Given a sequence (a_n) with $a_n = \cos(\frac{n\pi}{3})$,
 - a) find three different subsequences, which converge to three distinct real numbers,
 - b) show that the sequence (a_n) is divergent.
2. Let (a_n) be a sequence with $a_1 = 1$ and $a_{n+1} = \sqrt{1 + a_n^2}$. Show that the sequence (a_n) is unbounded and thus divergent.
(Hint: List first few terms to see the patten and show the result.)
3. Suppose that $a_1 > 0$ and $a_{n+1} = \sqrt{1 + a_n}$. Show that (a_n) is a Cauchy sequence.
4. Use definition to show that $a_n = (-1)^n + \frac{1}{n}$ is not a Cauchy sequence.
5. Let (a_n) be a sequeunce with $a_1 = 1$ and $a_{n+1} = \sqrt{3a_n + 4}$. Show that the sequence (a_n)
 - a) is bounded with $0 < a_n < 4$,
 - b) is convergent.