PRACTICE FINAL
(SOLNS [redacted])

MULTIPLE CHOICE 10 problems, 5 pts each

1) The equation of the line joining $\overset{x_1\ y_1}{(3,-2)}$ and $\overset{x_2\ y_2}{(1,6)}$ is

a) $y = x+5$

b) $y+2 = \frac{1}{6}(x-3)$

c) $y = 10-4x$

d) $3x-2y=6$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{8}{-2} = -4$$

$$y-(-2) = -4(x-3)$$

$$y = -4x+12-2 = 10-4x$$

2) $\lim\limits_{x\to\infty} \frac{x}{x^2+1} =$

a) 0

b) $-\infty$

c) ∞

d) 1

divide by highest power in denom

$$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{1}{x^2}} = \frac{0}{1} = 0$$

3) $\lim\limits_{x\to\infty} \frac{2x^2+3x+1}{4x^2-2} =$

a) $-\infty$

b) ∞

c) $\frac{1}{2}$

d) $2/3$

same as above $\dfrac{2+3/x+\frac{1}{x^2}}{4-2/x^2} = 2/4$

4) $f(x) = \begin{cases} x^2 & \text{if } x>1 \\ \frac{cx}{2} & \text{if } x\le 1 \end{cases}$ what c value makes f(x) continuous

a) 3

b) 0

c) NONE OF ABOVE

d) 2

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 = 1 \text{ so } \lim_{x\to 1^-} \frac{cx}{2} = 1 \iff c = 2$$

5) $\int_0^1 x+e^x =$

a) e

b) DNE

c) $1-e$

d) $e-\frac{1}{2}$

$$= \Big|_0^1 \frac{x^2}{2}+e^x = \left(\tfrac{1}{2}+e\right)-(0+1)$$

$$= e-\tfrac{1}{2}.$$

6) $\int_2^3 f(x)dx = 3$, $\int_3^5 \frac{f(x)}{2}dx = 6$. Then $\int_2^5 f(x)dx =$

a) CAN'T BE DET'D FROM INFO
b) 9
c) 3
d) 15

$\int_2^3 f(x) = 3 + \int_3^5 f(x) = 12 = 15 = \int_2^5 f(x)$

7) The derivative of $\ln(2x^3+1) =$

a) e^{2x^3+1}
b) $(\ln(2x^3+1)) \cdot 6x^2$
c) $\frac{6x}{2x^3+1}$
d) $\frac{6x^2}{2x^3+1}$

$\frac{d(\text{inside})}{\text{inside}} = \frac{6x^2}{2x^3+1}$

8) Find eqn of line thru (1,2), $\perp$ to the line $y = 2x+2$

a) $y-1 = 2(x-2)$
b) $y-1 = -2(x-2)$
c) $y-2 = -\frac{1}{2}(x-1)$
d) $y-2 = \frac{1}{2}(x-1)$

slope of $\perp$ = - reciprocal $= -\frac{1}{2}$

So $(y-2) = -\frac{1}{2}(x-1)$

9) Bacteria grows at a rate proportional to amount present, doubling every minute. If there are 10 at $t=0$ and 20 at $t=1$, find the amount at time t

a) $10e^t$
b) $10 \cdot 2^t$
c) $10e^{2t}$
d) $\frac{10}{e^2}e^{2t}$

$y = ce^{kt}$ $\quad 10 = ce^{k \cdot 0} = c \Rightarrow c = 10$, $\quad 20 = 10e^{k \cdot 1} \Rightarrow 2 = e^k$

$\ln 2 = k$

So $y = 10e^{(\ln 2)t} = 10 \cdot 2^t$

10) If $f(x,y) = x^2 + \frac{y}{x}$, then $f_x =$

a) $2x + \frac{x \cdot y' - y}{x^2}$
b) $2x - y/x^2$
c) $x^2 + \ln(y/x)$
d) $2x + \ln(y/x)$

$f_x = 2x + \frac{x \cdot 0 - y \cdot 1}{x^2}$

WORKOUT PROBS

(1)

$$f' = \frac{(x+1)^2 \cdot 1 - x \cdot 2(x+1)}{(x+1)^4} \leftarrow = x^2+2x+1-2x^2-2x = -x^2+1 = f'$$

$(x+1)^4$ ← always >0 if $x \neq -1$

(a)

↘	$\boxed{\text{DNE}}$	↗	0	↘
sign f': $-$	-1	$+$	$+1$	$-$

$1-x^2 = (1+x)(1-x)$

$\Rightarrow f' = \frac{1-x}{(1+x)^3}$

0 at $x=1$, DNE $x=-1$

$$f' = \frac{(1-x)(1+x)}{(1+x)^4} = \frac{(1-x)}{(1+x)^3}, \text{ so } f'' = \frac{(1+x)^3(-1) - (1-x)\cdot 3(1+x)^2}{(1+x)^6}$$

$(1+x)^6$ ← always $+$

(b)

$$\text{top} = (1+x)^2\left(-(1+x) - 3(1-x)\right)$$
$$= (1+x)^2 (2x-4)$$

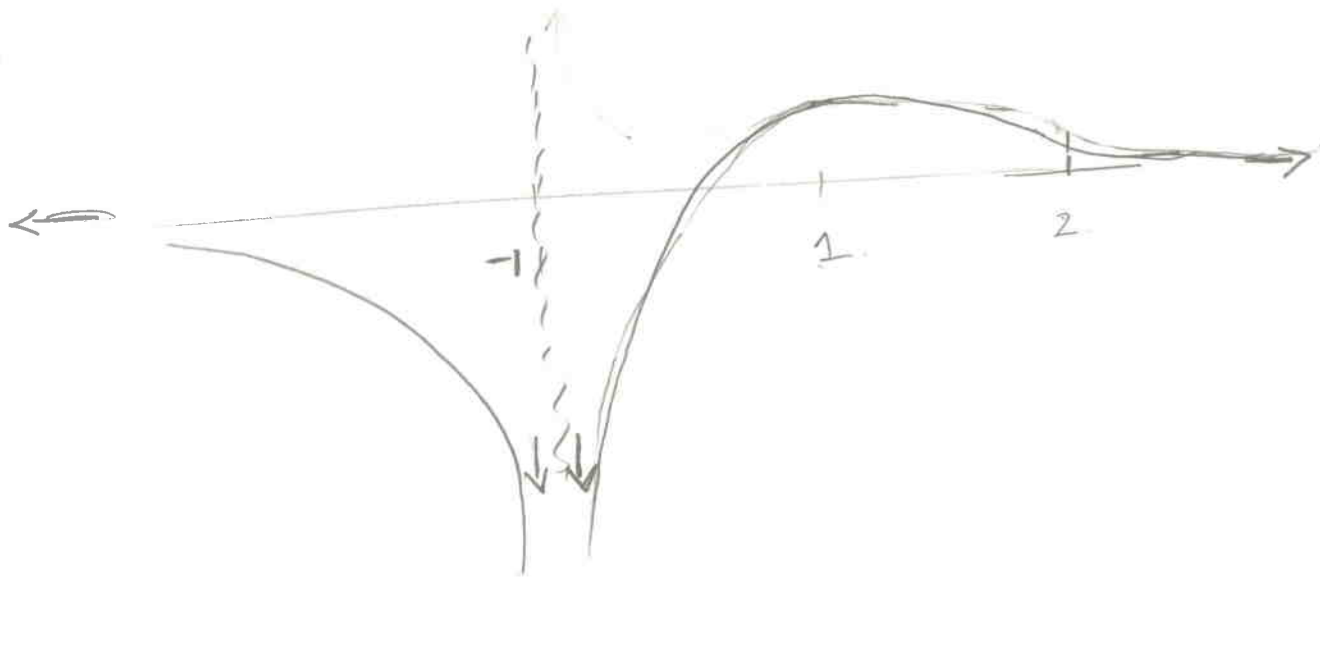
$(1+x)^2$: always $+$

$(2x-4)$ ← zero when $x = 2$

$\cap$		$\cup$
sign f'': $-$	2	$+$

(c) $\lim_{x \to -1^+} f(x) = -\infty$, $\lim_{x \to -1^-} f(x) = -\infty$, $\lim_{x \to +\infty} = \lim_{x \to -\infty} = 0$

(d)



(2)

a) $u = x^3+1$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$\rightsquigarrow \int x^2 (x^3+1)^{3/4}\,dx$

$= \int u^{3/4} \cdot \frac{1}{3}\,du$

$\frac{1}{3} \cdot \frac{4}{7} u^{7/4} = \frac{4}{21}(x^3+1)^{7/4} + C$

b) $u = (\ln(x))^2$ $\quad du = 2\ln(x) \cdot \frac{1}{x}$

$dv = x\,dx$ $\quad V = x^2/2$

$\int u\,dv = uv - \int v\,du = \frac{x^2}{2} \cdot (\ln(x))^2 - \underbrace{\int \frac{x^2}{2} \cdot 2\ln x \cdot \frac{1}{x}\,dx}$

$= - \underbrace{\int x \ln x\,dx}_{\text{second } \int \text{ by parts}}$

$u = \ln x \quad du = \frac{dx}{x}$

$dv = x\,dx \quad V = \frac{x^2}{2}$

so $\int x \ln x\,dx$

$= \frac{x^2}{2}\ln x - \underbrace{\int \frac{x^2}{2} \cdot \frac{dx}{x}}$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4}$

$\frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C.$

(3)

a) $c' = \frac{2}{5}x + 4$, $c'(4) = \frac{8}{5} + 4 = \frac{28}{5}$ = slope

when $x = 4$, $y = \frac{1}{5}(4)^2 + 16 + 57 = \frac{16}{5} + 57$ so

eqn of line is

$$y - \left(\frac{16}{5} + 57\right) = \frac{28}{5}(x - 4)$$

b) marg cost $= c' = \boxed{\frac{2}{5}x + 4}$

$R(x) = xP(x) = \frac{x}{4}(36 - x)$ so $R' =$ marg rev $=$

$\frac{1}{4}(36 - x) - \frac{x}{4} = \boxed{9 - \frac{x}{2}}$

c) $c'(4) = \frac{2}{5}(4) + 4 = \frac{28}{5}$

$$\begin{aligned} C(5) &= 5 + 20 + 57 \\ -C(4) &= -\left(\frac{16}{5} + 16 + 57\right) \\ \hline & \frac{9}{5} + 4 = \frac{29}{5} \end{aligned}$$

slope of tan line at 4 $= c'(4)$ is a very good approx $\left(\frac{28}{5}\right)$ to actual cost of 5^{th} unit $\left(\frac{29}{5}\right)$.

(4) $f_x = -16x^{-2} + 2x$

$f_y = -6y^{-2} - 6y$

$2x = \frac{16}{x^2} \quad 2x^3 = 16, \; x^3 = 8, \; x = 2$

$-6\left(y + \frac{1}{y^2}\right) = 0 \Rightarrow y = \frac{-1}{y^2} \quad y^3 = -1, \; y = -1$

crit pt at $(2,-1)$

Max or min if $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$ is + and

$\downarrow$ Max: $f_{xx} < 0$ $\quad$ $\downarrow$ min: $f_{xx} > 0$

Saddle if $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$ is −

$f_{xx} = 2 + 32x^{-3}$

$f_{yy} = -6 + 12y^{-3} \longrightarrow$ at $(2,-1)$, $\begin{vmatrix} 2 + \frac{32}{3^3} & 0 \\ 0 & -6 + \frac{12}{(-1)^3} \end{vmatrix} = -$

$f_{xy} = 0$

so saddle at $(2,-1)$

5 a) $E(p) = \frac{p}{q} \cdot \frac{dq}{dp}$ For $q = 10 - p$

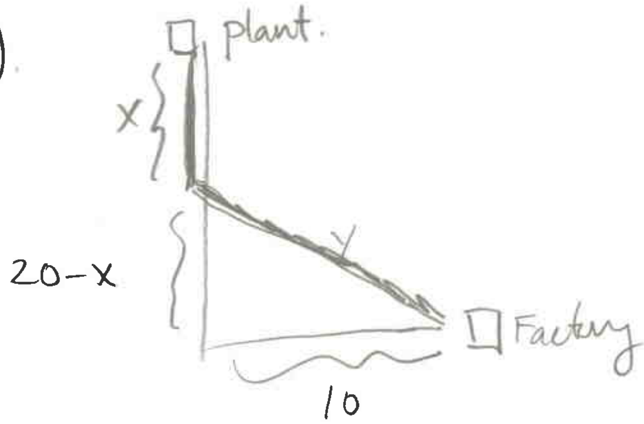
$dq = -dp$ so $\frac{dq}{dp} = -1$

$= \frac{p}{10-p} \cdot (-1) = \boxed{\frac{p}{p-10}}$

$E(5) = \frac{5}{5-10} = \frac{5}{-5} = -1$

So $|E(5)| = 1 \Rightarrow$ unit elasticity

6)



$y = \sqrt{10^2 + (20-x)^2} = \sqrt{100 + 400 - 40x + x^2}$

$y = \sqrt{x^2 - 40x + 500}$

$C(x) = 1 \cdot x + 10 \cdot \sqrt{x^2 - 40x + 500}$

$C'(x) = 1 + 10 \cdot \frac{1}{2}(x^2 - 40x + 500)^{-1/2} \cdot (2x - 2)$

$C'(x) = 1 + \frac{10(x-1)}{\sqrt{x^2 - 40x + 500}}$

Find when $C'(x) = 0$, check if min / max