

Math 225
Review Sheet for Final Exam
May 5, 2009

1. Linear System of Equations. Know how to:

- write them in vectorial and matrix form, see Sections 1.3 and 1.4;
- solve them using:
 - row reduction, see Section 1.2;
 - inverse of the matrix of the system (works only when the number of the unknowns equals the number of equations), see Section 2.2;
 - Cramer rule (works only when the number of the unknowns equals the number of equations), see Section 3.3;
- write their solutions in (parametric) vector form, see Section 1.5.

You should also know when the linear system of equations (in matrix form) $Ax = b$:

- has no solution (inconsistent), or, on the contrary, has at least one solution (consistent) and how are these situations related to the span of the columns of A , or to $\text{Col}A$, see Sections 1.2, 1.4 and 4.2;
- has at most one solution, or, on the contrary, infinitely many solutions and how are these situations related to: the linear independence/dependence of the columns of A , or $\text{Nul}A$, or the dimension $\text{Nul}A$, see Sections 1.7, 4.2, 4.5.

2. Matrix Operations and Invertible Matrices. Know:

- how to add two matrices, how to multiply a matrix by a number (scalar), how to multiply two matrices, how to take the transpose of a matrix, and the properties of all these operations, see Section 2.1;
- the definition of an invertible matrix, see Section 2.2;
- how to use invertible matrices to solve systems of equations, see Theorem 5 in Section 2.2;
- how to compute the inverse of a matrix:
 - for a 2×2 matrix, see Theorem 4 in Section 2.2;
 - via row operations (row reduction), see Section 2.2;
 - using determinants, see Section 3.3;
- properties of the inverse of a matrix, see Theorem 6 in Section 2.2;
- how to characterize the invertibility of a square matrix A via:
 - linear independence of columns of A , or the number of solutions of $Ax = 0$, or the $\text{Nul}A$, or the dimension of $\text{Nul}A$, see Theorem 8 in Section 2.3 and Theorem in Section 4.6;
 - consistency of $Ax = b$, or the span of columns of A , or $\text{Col}A$, or the dimension of $\text{Col}A$, or $\text{rank}A$, see Theorem 8 in Section 2.3 and Theorem in Section 4.6;
 - the determinant of A , see Theorem 4 in Section 3.2.
 - the invertibility of the transpose of A , see Theorem 8 in Section 2.3.

3. Determinants. Know:

- the definition of the determinant of a 1×1 matrix, 2×2 matrix, larger matrices via cofactor expansions, see Sections 3.1;
- how a determinant changes after a row operation or when taking the transpose, or after multiplying two matrices, see Section 3.2;

- how to calculate determinants using row, column operations and cofactor expansions, in particular how to compute determinants of upper or lower triangular matrices;
- how to use determinants to:
 - establish whether a matrix is invertible, see Theorem 4 in Section 3.2;
 - solve linear system of equations using the Cramer rule, see Section 3.3;
 - calculate the inverse of a matrix, see Section 3.3;
 - calculate the area of a parallelograms or volumes of parallelepipeds, see Section 3.3;

4. Vector Spaces. Know:

- the definition of a vector space, properties of the operations in a vector space, definition of span, see Section 4.1;
- the definition of a subspace, how to check whether given sets are subspaces in particular why for an $m \times n$ matrix A , $\text{Nul}A$ and $\text{Col}A$ are subspaces of \mathbb{R}^n respectively \mathbb{R}^m , see Sections 4.1 and 4.2;
- the definition of linear dependence and linear independence of a set vectors and how to check whether a given set of vectors is linearly dependent/independent, in particular the relation between linear dependence and independence of vectors in \mathbb{R}^n and the number of solutions of the equation $Ax = 0$, see Sections 4.3 and 1.7;
- the definition of a basis in a vector space, the definition of the dimension of a vector space, how to find basis and the dimension of $\text{Nul}A$ and $\text{Col}A$ where A is an $m \times n$ matrix, see Section 4.3 and 4.5;
- the definition of the rank of a matrix, the Rank Theorem and its applications to systems of linear equations and to establishing the invertibility of a matrix, see Section 4.6;

5. Eigenvalues, Eigenvectors and Diagonalization of Square Matrices Know:

- the definition of an eigenvalue and an eigenvector, see Section 5.1;
- how to compute the eigenvalues and the corresponding eigenspaces for given square matrix, see section 5.2;
- when is a square matrix diagonalizable and how its eigenvalues and eigenvectors are used to diagonalize a matrix, see section 5.3;
- how to use diagonalization in computing the powers of a square matrix, see Section 5.3.

6. Inner Product, Distance, Orthogonality and Least Square Problems Know:

- the definition of inner product of vectors in \mathbb{R}^n and its properties, the definitions of the length of a vector and distance between two vectors, see Section 6.1;
- the definition of orthogonality and Pytagora's Theorem, see Section 6.1;
- the definition of orthogonal sets of vectors in \mathbb{R}^n and their linear independence, see Section 6.2;
- the definition and how to calculate the orthogonal projection of a vector onto a subspace of \mathbb{R}^n , see Section 6.3;
- the definition of least squares solutions and how to calculate them, see Theorem 13 in Section 6.5.