

Homework #10
Ch. 2.4 #8, 12, 14, 16

Chapter 2.4

#8

We produce the following matrix:

$$\left[\begin{array}{ccc|c} -1 & 5 & 4 & -3 \\ -2 & 6 & 4 & -8 \\ -1 & 1 & 1 & -4 \end{array} \right]$$

We use the entry in row 1, column 1 as our pivot, so we begin by multiplying by -1. We then add twice row 1 to row 2 and add row 1 to row 2.

$$\left[\begin{array}{ccc|c} 1 & -5 & -4 & 3 \\ 0 & -4 & -4 & -2 \\ 0 & -4 & -3 & -1 \end{array} \right]$$

We now use the entry in row 2, column 2 as our next pivot. We first multiply row 2 by $-\frac{1}{4}$, then we add 5 times row 2 to row 1 and add 4 times row 2 to row 3.

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{11}{2} \\ 0 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 1 & 1 \end{array} \right]$$

Our last pivot is in the third row and third column. We subtract row 3 from row 1 and row 2.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{9}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 1 \end{array} \right]$$

The solution is then $x_1 = \frac{9}{2}, x_2 = -\frac{1}{2}, x_3 = 1$.

#12

We begin with the matrix

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 5 \\ 3 & 8 & 2 & 4 & 17 \\ -2 & 0 & 3 & 5 & -1 \\ 1 & 0 & -5 & -11 & 14 \end{array} \right]$$

We use row 1, column 1 as our first pivot, subtracting 3 times row 1 from row 2, adding 2 times row 1 to row 3, and subtracting row 1 from row 4.

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 5 \\ 0 & 2 & 2 & 4 & 2 \\ 0 & 4 & 3 & 5 & 9 \\ 0 & -2 & -5 & -11 & 9 \end{array} \right]$$

Our next pivot is row 2, column 2. We subtract row 2 from row 1, subtract twice row 2 from row 3, and add row 2 to row 4.

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & -4 & 3 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & -1 & -3 & 5 \\ 0 & 0 & -3 & -7 & 11 \end{array} \right]$$

The next pivot is row 3, column 3. We multiply row 3 by -1, then add twice row 3 to row 1, subtract row 3 from row 2, and add three times row 3 to row 4.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -7 \\ 0 & 1 & 0 & -1 & -4 \\ 0 & 0 & 1 & 3 & -5 \\ 0 & 0 & 0 & 2 & -4 \end{array} \right]$$

Our next pivot is row 4, column 4. We multiply row 4 by $\frac{1}{2}$, then subtract twice row 4 from row 1, add row 4 to row 2, and subtract three times row 4 from row 3.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{array} \right]$$

We end up with the solution $x_1 = -3, x_2 = -6, x_3 = 1, x_4 = -2$.

#14

As usual, we start by writing the corresponding matrix.

$$\left[\begin{array}{ccccc|c} 1 & 5 & -3 & -2 & 3 & 5 \\ -2 & -10 & 7 & 6 & -9 & -12 \\ 2 & 10 & -8 & -9 & 8 & 6 \end{array} \right]$$

Our first pivot is row 1, column 1. We add twice row 1 to row 2, and subtract twice row 1 from row 3.

$$\left[\begin{array}{ccccc|c} 1 & 5 & -3 & -2 & 3 & 5 \\ 0 & 0 & 1 & 2 & -3 & -2 \\ 0 & 0 & -2 & -5 & 2 & -4 \end{array} \right]$$

We choose row 2, column 3 as our next pivot. We add three times row 2 to row 1 and add twice row 2 to row 3.

$$\left[\begin{array}{ccccc|c} 1 & 5 & 0 & 4 & -6 & -1 \\ 0 & 0 & 1 & 2 & -3 & -2 \\ 0 & 0 & 0 & -1 & -4 & -8 \end{array} \right]$$

Our last pivot is row 3, column 4. We first multiply by -1, then subtract four times row 3 from row 1 and subtract twice row 3 from row 2.

$$\left[\begin{array}{ccccc|c} 1 & 5 & 0 & 0 & -30 & -33 \\ 0 & 0 & 1 & 0 & -11 & -18 \\ 0 & 0 & 0 & 1 & 4 & 8 \end{array} \right]$$

We don't have a single solution to this system as not all of our variables are basic. We identify our basic variables as x_1, x_3, x_4 , leaving x_2 and x_5 as our free variables.

We set $x_2 = s$ and $x_5 = t$. Using the three rows to solve for each of the basic variables, we get $x_1 = -5s + 30t - 33$, $x_3 = 11t - 18$, and $x_4 = -4t + 8$.

#16

We produce the corresponding matrix:

$$\left[\begin{array}{cccc|c} 4 & 12 & 8 & 4 & 20 \\ 2 & 8 & 12 & 6 & 8 \\ 3 & 13 & 25 & 2 & 5 \end{array} \right]$$

Our first pivot is row 1, column 1, so we first divide row 1 by 4. We then subtract twice row 1 from row 2 and subtract three times row 1 from row 3.

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 1 & 5 \\ 0 & 2 & 8 & 4 & -2 \\ 0 & 4 & 19 & -1 & -10 \end{array} \right]$$

We choose row 2, column 2 as our next pivot. We first divide row 2 by 2, then subtract 3 times row 2 from row 1 and subtract 4 times row 2 from row 3.

$$\left[\begin{array}{cccc|c} 1 & 0 & -10 & -5 & 8 \\ 0 & 1 & 4 & 2 & -1 \\ 0 & 0 & 3 & -9 & -6 \end{array} \right]$$

Our final pivot is row 3, column 3, so we start by dividing row 3 by 3. We then add 10 times row 3 to row 1 and subtract 4 times row 3 from row 2.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -35 & -12 \\ 0 & 1 & 0 & 14 & 7 \\ 0 & 0 & 1 & -3 & -2 \end{array} \right]$$

We identify x, y, z as basic variable and w as free. We set $w = t$, then solve for the basic variables. We get $x = 35y - 12$, $y = -14t + 7$, and $z = 3t - 2$.