Math 124 C1 Homework #3
Ch. 5.3. #18, 24, 28, 32
Ch. 6.1. #16, 28, 36
Ch. 6.2. #2, 6, 10, 22, 34

Chapter 5.3

16.

Since we have roles for men and roles for women, we think of casting the whole play as an activity with two stages. The first stage is casting the men; since we are picking 5 men from a pool of 8 without replacement and with order mattering, the number of ways to do this is P(8,5). The second stage is picking 4 women from a pool of 9, with order and without replacement, can be done in P(9,4) ways.

Using the fundamental principle of counting, the total number of ways to cast the play is the product of the ways to cast the two stages: P(8,5)*P(9,4).

24.

This is an easy version of #28 below. BIB has letters in the following amount: B x2 and I x1.

The first stage is placing the two B's into the three spots available; there are C(3,2) ways to do this. The second stage is placing the I; since there is a single spot remaining, there is only one way to do this.

Thus there are C(3,2) words to write down using all letters from BIB.

28.

We break down MISSISSIPPI into its component letters: I x4, M x1, P x2, Sx4. Further, we're going to break this into stages, just as we did with ALABAMA. The stages will be placement of duplicated letters.

The first stage, then, is to place the four letters I. Since we are using all 11 letters, we need to select 4 spots in which to put I's. There are C(11,4) ways to do this.

The second stage is to place the four letters S. After placing the 4 I's, there are 7 spots remaining; there are then C(7,4) ways to select spots in which to put S's.

The third stage is to place the two P's. There are now 3 spots remaining, so there are C(3,2) ways to place the P's.

The fourth stage is to place the M. Since there is a single spot left for the M, there is only one way to place it.

Now the principle of counting tells us that we should multiply together the ways to do each stage to get the total number of ways to write down a word: C(11,4)$*$C(7,4)$*$C(3,2).

32.a.

This problem is essentially the same as the problem above, only instead of letters, we have colored buses. Since the buses of a given color are indistinguishable, it does not matter which blue bus is in a given spot, only that the bus there is blue.

So we have 4 red buses, 2 blue buses, and 1 white bus. We'll do this problem in three stages, each stage consisting of placing a single color of buses in the convoy.

There are 7 buses total and so there are 7 spots in our convoy. If we are to place the red buses first, there are C(7,4) ways to put four red buses in the seven spots.

Now we place the blue buses; since there are only 3 spots left, there are C(3,2) ways to place blue buses.

Now we are left to place the single white bus in the single remaining spot; there is only 1 way to do this.

Thus there are C(7,4)∗C(3,2) ways to send the buses out on a convoy.

32.b.

There are also C(7,4)∗C(3,2) ways to send 4 buses out on one route and 3 out on another. For each 7-bus convoy from part a, we split off the first four buses (keeping their order) from the remaining three buses. This gives us a 4- and 3-bus convoy pair, and there are then as many such pairs as there are 7-bus convoys to begin with.

Chapter 6.1

16.

Since Pr is a probability function, we know that

$$\begin{aligned}
\Pr(s_1) + \Pr(s_2) + \Pr(s_3) &= 1 \\
\frac{3}{7} + \frac{2}{7} + \Pr(s_3) &= 1 \\
\Pr(s_3) &= \frac{2}{7}
\end{aligned}$$

28.

We recall that since $E = \{s_1, s_2\}$, $\Pr(E) = \Pr(s_1) + \Pr(s_2) = 0.37 + 0.19 = 0.56$. Likewise, $\Pr(F) = \Pr(s_1) + \Pr(s_3) + \Pr(s_5) = 0.37 + 0.22 + 0.13 = 0.72$.

36.

We are rolling a single die 5 times, so our sample space consists of 5-tuples of numbers from 1 to 6: $S = \{(r_1, r_2, r_3, r_4, r_5) : each r_i = 1, 2, 3, 4, 5, 6\}$. The size of the sample space is 6^5.

Our event, E, is all outcomes in which there are exactly two fours. This event is far too large to write out all of its elements, so we count them using our tricks from Chapter 5.

If we think about writing down a 5-tuple that qualifies, we can write such a thing down in two stages. First, we decide which rolls turned up as 4's. There is no replacement and order does not matter, so there are C(5,2) ways to write down the 4's.

Next, we have to fill in the remaining 3 spots with numbers which are *not* 4's. That leaves 5 choices for each spot, so there are 5^3 ways to do this.

In total, then, $n(E) = C(5, 2) * 5^3$.

We now use our rule that $\Pr(E) = \frac{n(E)}{n(S)}$ to see that $\Pr(E) = \frac{C(5,2)*5^3}{6^5}$.

Chapter 6.2

2.

Since our experiment requires that we look at the suit and rank of our cards, our sample space will be $S = \{(s, r) : s = \text{any suit}, r = A, 2, 3, ..., 10, J, Q, K\}$. Let B be the event of drawing a black card; then $B = \{(s, r) : s = \text{spade, club}, r = A, 2, ..., 10, J, Q, K\}$. Let A be the event of drawing an ace, so that $A = \{(s, r) : s = \text{any suit}, r = A\}$.

The event of drawing a card which is black or an ace corresponds to the set $A \cup B$. To find $n(A \cup B)$, we use inclusion exclusion: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Since only the ace of spaces and ace of clubs are both black and an ace, $n(A \cap B) = 2$. Hence $n(A \cup B) = 4 + 26 - 2 = 28$.

Thus the probability of drawing a card that is black or an ace is $\frac{28}{52}$.

6.

Let S be the usual sample space for rolling two six-sided dice. Let $E_4 = \{(1, 3), (2, 2), (3, 1)\}$, $E_5 = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$, and $E_6 = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$ be the event sets for rolling a 4, 5, and 6, respectively.

Since these events are disjoint, $n(E_4 \cup E_5 \cup E_6) = 3 + 4 + 5 = 12$.

Thus the probability of rolling 4, 5, or 6 is $\frac{12}{36}$.

10.

The best way to do this problem is by negation. If E is the event of getting at least two sixes, then E' is the event of getting one or zero sixes.

Let E_0 be the event of getting no sixes. If we were to write down a sequence of dice rolls with no sixes, we see that for each die, we have a choice of 5 numbers which are not six to write down. Thus, $n(E_0) = 5^4$.

Let E_1 be the event of getting exactly 1 six. We first place the six in one of four places, then assign numbers which are not six to the remaining three places. There are thus $6 * 5^3$ ways to do this.

Since E_0 and E_1 are disjoint and $E' = E_0 \cup E_1$, $n(E') = 5^4 + 6 * 5^3$. Thus $\Pr(E') = \frac{5^4+6*5^3}{6^4}$, and so $\Pr(E) = 1 - \Pr(E') = 1 - \frac{5^4+6*5^3}{6^4}$.

22.a.

We use the data presented to develop a probability function. Let B be the set of roads with blue correct and R the set of roads with red correct. We have $n(B) = 692$ and $n(R) = 714$. Further, $n(R \cap B) = 623$. Thus, there are $n(R) - n(R \cap B) = 714 - 623 = 91$ roads with only red correct. Thus the probability of having red correct but not blue is $\frac{91}{750}$.

22.b.

Again, we notice that the number of roads with only blue correct is $n(B) - n(R \cap B) = 692 - 623 = 69$. Thus the probability of having blue correct but not red is $\frac{69}{750}$.

34.

This is a straightforward application of the probability version of inclusion-exclusion. Just as with sets, we have a three-set version of inclusion-exclusion:

$$\begin{aligned}
\Pr(E \cup F \cup G) &= \Pr(E) + \Pr(F) + \Pr(G) \\
&\quad - \Pr(E \cap F) - \Pr(E \cap G) - \Pr(F \cap G) \\
&\quad + \Pr(E \cap F \cap G) \\
&= 0.34 + 0.33 + 0.51 - 0.10 - 0.16 - 0.12 + 0.03 \\
&= 0.83
\end{aligned}$$