Math 124 C1 Homework #5
Ch. 7.1 #6, 11, 14, 20, 24, 28
Ch. 7.2 #4, 12, 14, 18, 22

Chapter 7.1

#6

Outcome	X	Y	$X - Y$	$X + Y$
1	1	3	-2	4
2	4	6	-2	10
3	9	9	0	18
4	16	12	4	28
5	25	15	10	40
6	36	18	18	54

#11

We make a table of the random variable X:

Outcome	X
1	2
2	1
3	6
4	3
5	10
6	5

Since the outcomes of rolling the die are all equally likely and each outcome results in a unique value for X, each value of X is equally likely. Therefore:

X	$\Pr(X)$
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$
10	$\frac{1}{6}$

#14

Each ball has an equal chance of being drawn, so let's count how many balls would give a particular value of X. If the red 1 ball is drawn, $X = 2$. If the red 2 ball is drawn, $X = 4$. If the black 2 ball is drawn, $X = 2$. If the black 3 ball is drawn, $X = 3$. If the black 4 ball is drawn, $X = 4$. Since two balls result in $X = 2$, $\Pr(X = 2) = \frac{2}{5}$. One ball results in $X = 3$, $\Pr(X = 3) = \frac{1}{5}$. Two balls result in $X = 4$, so $\Pr(X = 4) = \frac{2}{5}$.

#20

Since we have replacement, each drawing of a ball is independent from the other two. Let's practice using Bernoulli's equation. Since we are counting red balls, let's consider a red ball as a success. The probability of success is then $p = \frac{7}{9}$. Then:

$$\begin{aligned}
\Pr(X = 0) &= C(3,0)\left(\frac{7}{9}\right)^0\left(\frac{2}{9}\right)^3 = \frac{8}{729}\\
\Pr(X = 1) &= C(3,1)\left(\frac{7}{9}\right)^1\left(\frac{2}{9}\right)^2 = \frac{84}{729}\\
\Pr(X = 2) &= C(3,2)\left(\frac{7}{9}\right)^2\left(\frac{2}{9}\right)^1 = \frac{294}{729}\\
\Pr(X = 3) &= C(3,3)\left(\frac{7}{9}\right)^3\left(\frac{2}{9}\right)^0 = \frac{343}{729}
\end{aligned}$$

#24

We have $n = 15$. If we need at least 8 successes, then we cannot have 7 or fewer successes. Hence we look in the table for:

$\Pr(\geq 8 \textit{ successes}) = 1 - \Pr(\leq 7 \textit{ successes}) = 1 - B(7; 15, 0.15) = 1 - 0.996 = 0.004$

#28

Since there are 16 bearings in a box, $n = 16$. Let's let a success be finding a bad bearing; then $p = 0.15$. The complement of the event of having at least 5 bad bearings is the event of having at most 4 bad bearings. Therefore,

$\Pr(\geq 5 \textit{ bad}) = 1 - \Pr(\leq 4 \textit{ bad}) = 1 - B(4; 16, 0.15) = 1 - .9209 = .0791$

Ch. 7.2

#4

Using the definition of expected value,

$$E(X) = -15 * 0.1 + -5 * 0.3 + 10 * 0.4 + 15 * 0.2 \qquad = -1.5 + -1.5 + 4 + 3 = 4$$

#12

Since we have pennies, nickels, dimes, and one quarter available, we should calculate the numerical value of any two of these coins, and then find the probability of each of the corresponding outcomes.

Outcome	X	Pr
(p,p)	2	$\frac{2}{12} * \frac{1}{11} = \frac{2}{132}$
(p,n) or (n,p)	6	$\frac{2}{12} * \frac{3}{11} + \frac{3}{12} * \frac{2}{11} = \frac{12}{132}$
(p,d) or (d,p)	11	$\frac{2}{12} * \frac{6}{11} + \frac{6}{12} * \frac{2}{11} = \frac{24}{132}$
(p,q) or (q,p)	26	$\frac{2}{12} * \frac{1}{11} + \frac{1}{12} * \frac{2}{11} = \frac{4}{132}$
(n,n)	10	$\frac{3}{12} * \frac{2}{11} = \frac{6}{132}$
(n,d) or (d,n)	15	$\frac{3}{12} * \frac{6}{11} + \frac{6}{12} * \frac{3}{11} = \frac{36}{132}$
(n,q) or (q,n)	30	$\frac{3}{12} * \frac{1}{11} + \frac{1}{12} * \frac{3}{11} = \frac{6}{132}$
(d,d)	20	$\frac{6}{12} * \frac{5}{11} = \frac{30}{132}$
(d,q) or (q,d)	35	$\frac{6}{12} * \frac{1}{11} + \frac{1}{12} * \frac{6}{11} = \frac{12}{132}$

Thus

$$\begin{aligned} E(X) &= 2 * \frac{2}{132} + 6 * \frac{12}{132} + 11 * \frac{24}{132} + 26 * \frac{4}{132} + 10 * \frac{6}{132} + 15 * \frac{36}{132} + \\ &= 30 * \frac{6}{132} + 20 * \frac{30}{132} + 35 * \frac{12}{132} \\ &= \frac{2244}{132} = 17 \end{aligned}$$

#14

If we guess randomly on a 3-choice multiple-choice question, we will get the question right $\frac{1}{3}$ of the time. Therefore, we can expect to get $120 * \frac{1}{3} = 40$ questions right on the whole exam by guessing.

#18a

We must go back to using Table A in the appendix to answer this question. Here, $n = 16$ and the probability of success is the probability of answering a question correctly: $p = 0.25$.

With these parameters, the probability of getting exactly 4 correct answers is $B(4) - B(3) = 0.6302 - 0.4050 = 0.2252$.

#18b

Getting 3, 4, or 5 correct answers has the probability $B(5) - B(2) = 0.8103 - 0.1971 = 0.6132$.

#22a

Since $\frac{4}{52} = \frac{1}{13}$ of the cards are aces, we would expect $5 * \frac{1}{13} = \frac{5}{13}$ cards to be aces.

#22b

Since $\frac{13}{52} = \frac{1}{4}$ of the cards are spades, we would expect $5 * \frac{1}{4} = \frac{5}{4}$ cards to be spades.