Math 124 C1 Homework #7
Ch. 1.3: #24, 26, 30, 32
Ch. 1.4: #14, 24, 28, 30

Chapter 1.3

#24

We begin by solving the second and third equations; this is easiest because the second has a $-6y$ term and the third has a $6y$ term, so we can add them together and eliminate y.

We do so, and get that $-x = 1$, giving that $x = -1$.

Plugging $x = -1$ into the second equation, we get that $-7 - 6y = 3$, which we solve to find that $y = \frac{-10}{6} = -\frac{5}{3}$.

We then must check whether the point $(-1, -\frac{5}{3})$ satisfies the first equation. By plugging in the corresponding values, we see that $4(-1) - 3(-\frac{5}{3}) = -4 - 5 = 1$, so $(-1, -\frac{5}{3})$ is the solution to this system.

#26

We first solve the subsystem of equations that do not involve k. By adding the first and second equations together, we see that $3x = 6$, so $x = 2$. Plugging this into the first equation, we get that $2 + y = 1$, so $y = -1$.

We then plug this solution to the first two equations into the third and solve for k: $2 - k = -1$, so $k = 3$. Thus this system of equations is consistent only for the value $k = 3$.

#30

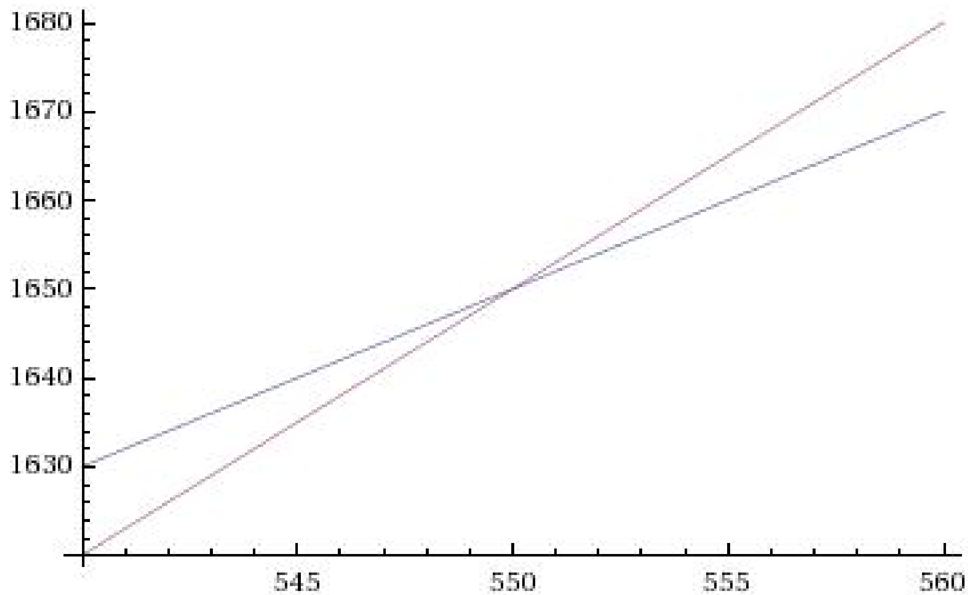
The information in the problem yields the following system of equations:

$$y = 550 + 2x$$
$$y = 3x$$

Here, the first equation is an expression for the cost y of buying x pounds of candy, since we have \$550 in base operating costs and \$2 per pound of candy. The second equation gives the income y from selling x pounds of candy at \$3 per pound.

Subtracting the second equation from the first gives $0 = 550 - x$, or $x = 550$.

So the breakpoint is $x = 550$, with profit $y = 1650$.



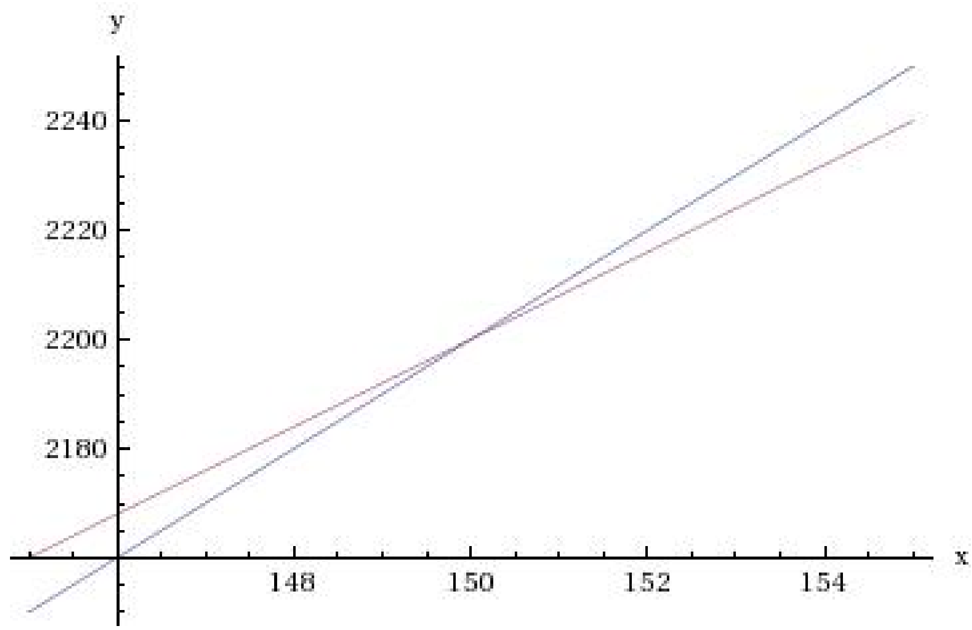
#32

Again, we set up the obvious system of equations.

$$y = 700 + 10x$$
$$y = 1000 + 8x$$

Subtracting equation 1 from equation 2, we get $0 = -300 + 2x$, hence $x = 150$ is the breakpoint. The cost at this point is 2200.

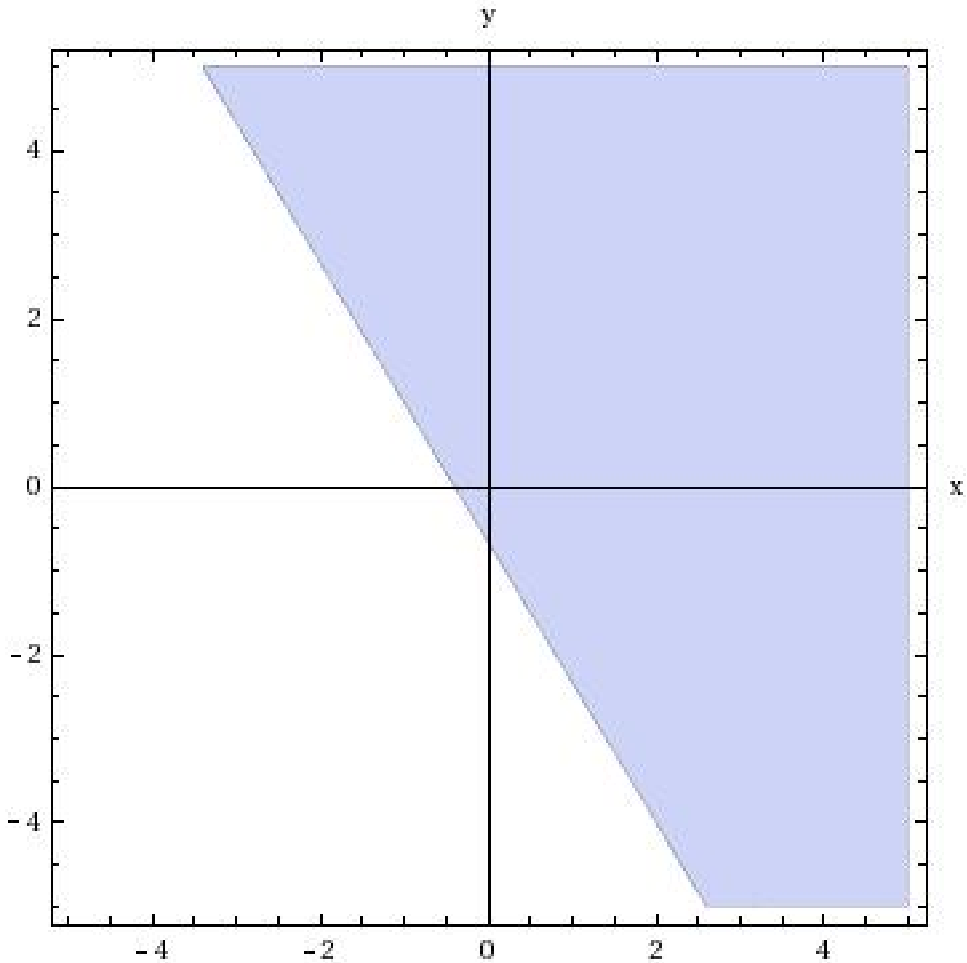
Graphing the two gives:



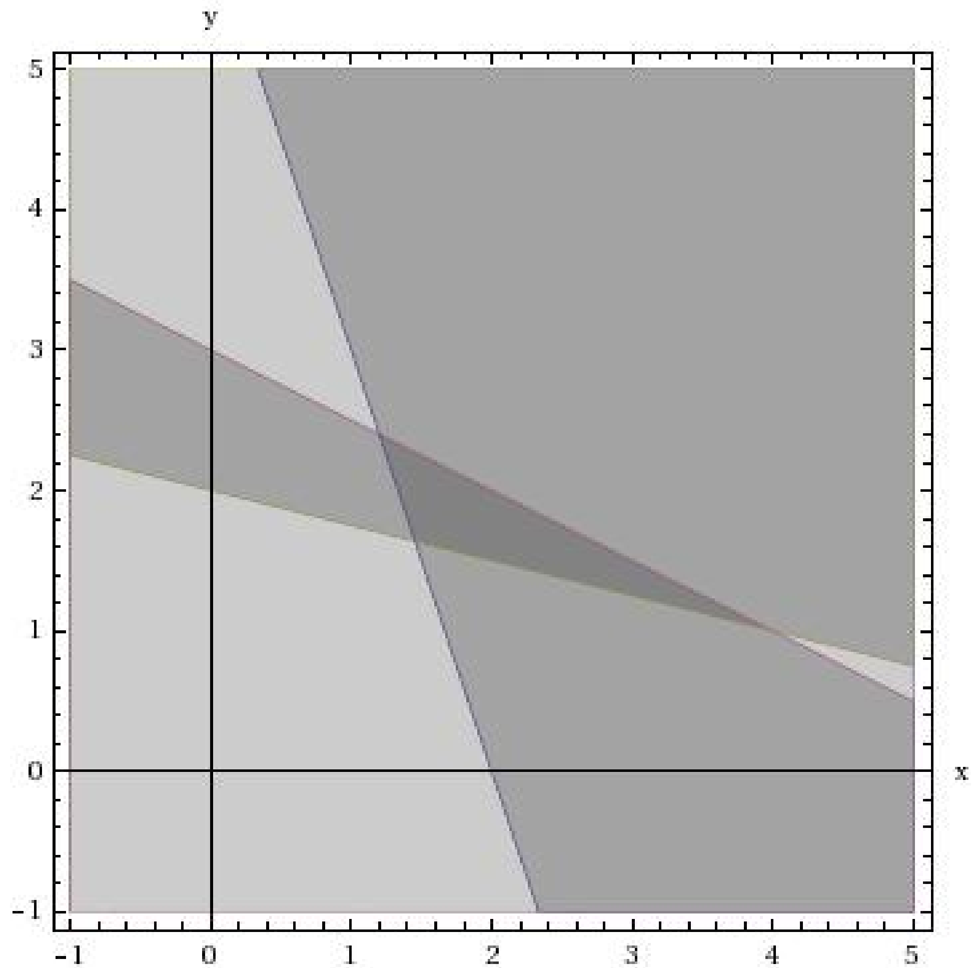
We can see that, for orders to the right of the break point, such as for 950 tires, the smaller slope line is cheaper, which corresponds to Mold B.

Chapter 1.4

#14



#24



The solution set is the darkest region on the above graph. Since it is a triangle we must find three corner points; they are each the intersection of two of the three lines whose equations we have.

Solving $3x + y = 6$ and $x + 2y = 6$, we get $5x = 6$, so $x = \frac{6}{5}$. Plugging back in, we get $y = \frac{12}{5}$, so one corner point is $(\frac{6}{5}, \frac{12}{5})$.

Solving $3x + y = 6$ and $x + 4y = 8$, we get $-11y = -18$, so $y = \frac{18}{11}$. Plugging back in, we get $x = \frac{16}{11}$, so another corner point is $(\frac{16}{11}, \frac{18}{11})$.

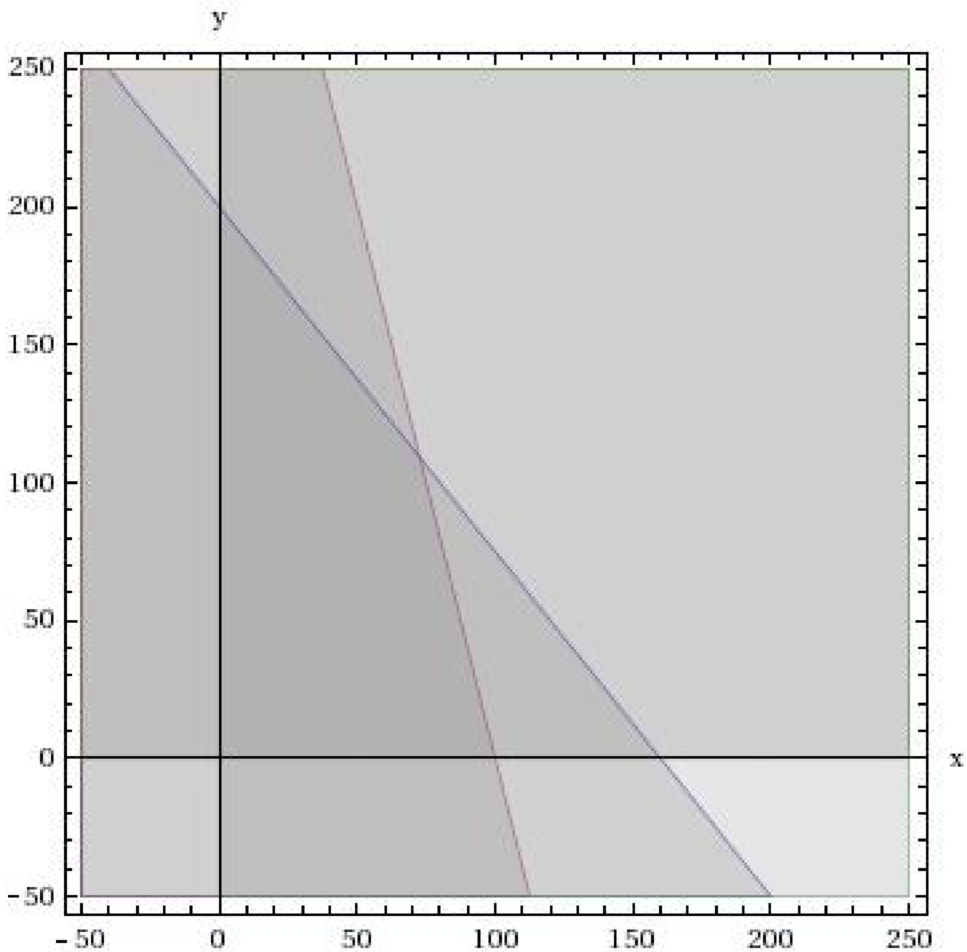
Lastly, solving $x + 2y = 6$ and $x + 4y = 8$, we get $y = 1$, and then $x = 4$, so $(4, 1)$ is the final corner point.

#28

We graph the following system of inequalities, where x is the number of Alpine shingles and y is the number of Green Meadow shingles.

$$250x + 200y \leq 40000$$
$$100x + 25y \leq 10000$$

We also include the constrants that $x \geq 0$ and $y \geq 0$, since we we can only manufacture nonnegative amounts of the two items.



The solution set is again the darkest-colored region above.

To get the corner point, we replace all inequalities with equalities. Multiplying the second equation by -8 and adding the two together yields $-550x = -40000$, so $x = -\frac{40000}{550} = -\frac{800}{11} =$ 72.7.

Plugging in x to the first equation gives that $y = \frac{1200}{11} =$ 109.1.

We see that $(0, 0)$ is also a corner point, and by observing the graph, we have an x-intercept and a y-intercept as corner points. It is easy to see that these are $(0, 200)$ and $(100, 0)$.

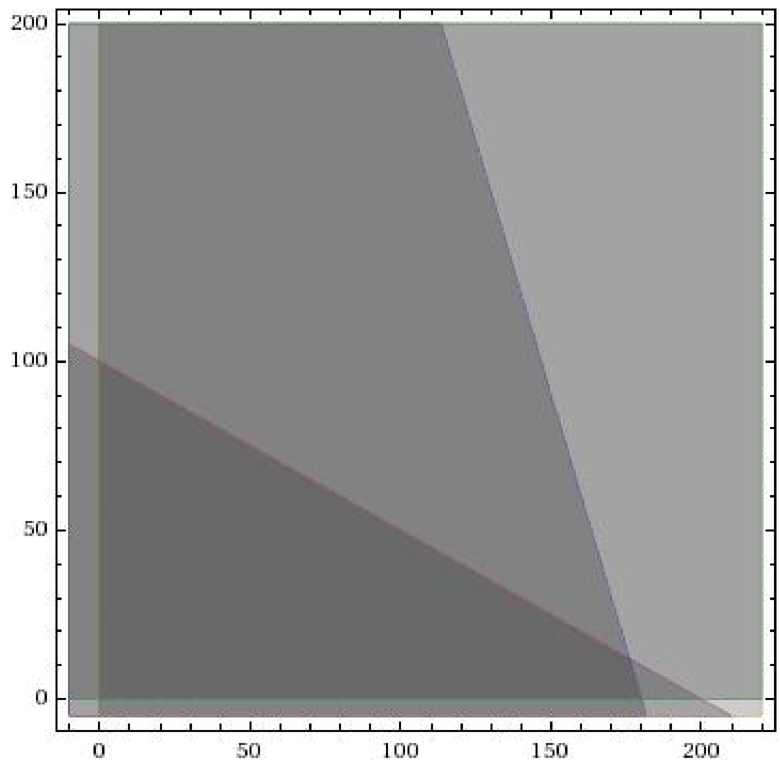
#30

Let x be the number of Bucking Bronco toys, and y the number of Locomotive toys. Again, we write down a system based on how much scrap and how much grade A stock each toy consumes:

$$\begin{aligned} 3x + y &\leq 540 \\ 2x + 4y &\leq 400 \end{aligned}$$

We should also impose the inequalities $x \geq 0$ and $y \geq 0$, again, because we can only manufacture a nonnegative number of toys.

We first graph the solution sets by graphing the corresponding lines and determining which half-planes are solution sets for each inequality.



The solution set is the darkest-colored region in the above graph.

Again, to find the first corner point, we need only find the point of intersection of the lines. By replacing the inequalities with equalities, multiplying the first by -4, and adding, we see that $-10x = -1760$, so $x = 176$.

Plugging in this value for x and solving for y, we get $528 + y = 540$, so $y = 12$.

The remaining corner points are x- and y-intercepts of lines; we can see that they are (0,100) and (180,0).