Math 124 C1 Homework #8

Ch. 2.1 #1,5,24,30,34

Note! If you are able to solve #34 and understand how to translate back and forth between systems of equations and matrices, you should be able to do well on the quiz this week. Be sure to practice these types of problems beyond what I have assigned on this homework, though!

Chapter 2.1

#1

The matrix corresponding to this system is:

$$\left[\begin{array}{ccc|c} 4 & -3 & 1 & 5 \\ 1 & 5 & -2 & -7 \\ -5 & 7 & -6 & 9 \end{array}\right]$$

#5

The matrix corresponding to this system is:

$$\left[\begin{array}{cccc|c} 4 & 2 & 3 & -1 & 10 \\ 8 & 5 & -2 & 4 & -2 \\ 7 & 4 & 5 & 3 & 4 \\ 2 & -1 & 3 & 1 & -5 \end{array}\right]$$

#24

This matrix is *not* in row echelon form; the third row's leftmost nonzero entry is not equal to one. To fix this, we multiply the third row by $\frac{1}{2}$ to get:

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & \frac{5}{2} \end{array}\right]$$

#30

We start with

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & -2 \\ 3 & 8 & -4 & -6 \\ -1 & 4 & -7 & 7 \end{array}\right]$$

.

We see that the first row has a leftmost nonzero entry equal to one, so we will cancel the rest of the numbers in the first column using this row. We perform the operations $R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 + R_1$ to get:

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & -2 \\ 0 & -1 & 2 & 0 \\ 0 & 7 & -9 & 5 \end{array}\right]$$

We see that row 2 has a leftmost nonzero entry -1, so we multiply by -1.

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & -2 \\ 0 & 1 & -2 & 0 \\ 0 & 7 & -9 & 5 \end{array}\right]$$

We now use the operation $R_3 \to R_3 - 7R_2$ to get:

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & -2 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 5 & 5 \end{array}\right]$$

Finally, we use the operation $R_3 \to \frac{1}{5}R_3$ to get:

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & -2 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

We notice that every 1 has zeros below it, that the leftmost entry in every nonzero row is 1, so this matrix is in row echelon form.

Using back substitution, we get that $x_3 = 1$.

The second row tells us that $x_2 + -2(1) = 0$, so $x_2 = 2$.

Lastly, the first row tells us that $x_1 + 3(2) - 2(1) = -2$, so $x_1 = -6$.

#34

We are given the matrix

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 3 & -24 \\ 2 & 7 & 11 & 7 & -54 \\ -1 & -1 & 0 & -7 & -6 \\ -2 & -9 & -14 & -5 & 74 \end{array}\right]$$

Since the first row has a leading 1, we will work with this row first. We perform operations $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + R_1$, and $R_4 \to R_4 + 2R_1$.

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 3 & -24 \\ 0 & 1 & 1 & 1 & -6 \\ 0 & 2 & 5 & --4 & -30 \\ 0 & -3 & -4 & 1 & 26 \end{array}\right]$$

Happily, we now have a leading one in the second row also, so we will use that to cancel the numbers below it in the second column with the operations $R_3 \to R_3 - 2R_2$ and $R_4 \to R_4 + 3R_2$.

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 3 & -24 \\ 0 & 1 & 1 & 1 & -6 \\ 0 & 0 & 3 & -6 & -18 \\ 0 & 0 & -1 & 4 & 8 \end{array}\right]$$

Now, we have the last two rows remaining. Because Row 4 has a leftmost nonzero entry -1, we'll do $R_4 \to -R_4$ and then $R_3 \leftrightarrow R_4$.

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 3 & -24 \\ 0 & 1 & 1 & 1 & -6 \\ 0 & 0 & 1 & -4 & -8 \\ 0 & 0 & 3 & -6 & -18 \end{array}\right]$$

Now we perform $R_4 \to R_4 - 3R_1$.

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 3 & -24 \\ 0 & 1 & 1 & 1 & -6 \\ 0 & 0 & 1 & -4 & -8 \\ 0 & 0 & 0 & 6 & 6 \end{array}\right]$$

Lastly, we give Row 4 a leftmost one by $R_4 \to \frac{1}{6}R_4$.

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & 3 & -24 \\ 0 & 1 & 1 & 1 & -6 \\ 0 & 0 & 1 & -4 & -8 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

Using back substitution, we first note that $x_4 = 1$.
This gives that $x_3 - 4(1) = -8$, so $x_3 = -4$.
Row 2 says that $x_2 - 4 + 1 = -6$, so $x_2 = -3$.
Lastly, we have $x_1 + 3(-3) + 5(-4) + 3(1) = x_1 - 9 - 20 + 3 = x_1 - 26 = -24$, so that $x_1 = 2$.