Math 124 C1 Homework #9

Ch. 2.2 #6, #8, #14

Ch. 2.3 #4, #6, #10, #18, #20, #22,

Chapter 2.2

#6

$$\left[\begin{array}{cccc|c} 1 & -2 & 4 & 0 & 3 \\ 0 & 1 & -1 & 2 & -1 \\ 0 & 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$R_2 \to R_2 - 2R_4$ and $R_3 \to R_3 + 3R_4$.

$$\left[\begin{array}{cccc|c} 1 & -2 & 4 & 0 & 3 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$R_1 \to R_1 - 4R_3$ and $R_2 \to R_2 + R_3$.

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$R_1 \to R_1 + 2R_2$.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

Thus, the solution is $x_1 = -1, x_2 = 2, x_3 = -1, x_4 = -2$.

#8

We produce the matrix:

$$\left[\begin{array}{ccc|c} 3 & 9 & -2 & 16 \\ 1 & 4 & 1 & 2 \\ 2 & 6 & -1 & 12 \end{array}\right]$$

$R_1 \leftrightarrow R_2$.

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 3 & 9 & -2 & 16 \\ 2 & 6 & -1 & 12 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$ and $R_3 \to r_3 + -2R_1$.

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & -3 & -5 & 10 \\ 0 & -2 & -3 & 8 \end{array}\right]$$

$R_3 \to -\frac{1}{2}R_3$, then $R_2 \leftrightarrow R_3$.

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & 1 & \frac{3}{2} & 4 \\ 0 & -3 & -5 & 10 \end{array}\right]$$

$R_3 \to r_3 + 3R_2$.

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & 1 & \frac{3}{2} & 4 \\ 0 & 0 & -\frac{1}{2} & -2 \end{array}\right]$$

$R_3 \to -2R_3$.

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 2 \\ 0 & 1 & \frac{3}{2} & 4 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \to r_1 + -1R_3$ and $R_2 \to r_2 + -\frac{3}{2}R_3$.

$$\left[\begin{array}{ccc|c} 1 & 4 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \to r_1 + -4R_2$.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Thus the solution is $x_1 = 6, x_2 = -2, x_3 = 4$.

0.1 #14

$$\left[\begin{array}{ccccc} 1 & 2 & -3 & -1 & 2 \\ 3 & 5 & 3 & 2 & -3 \\ -5 & -9 & 8 & 5 & -16 \end{array}\right]$$

$R_2 \to r_2 + -3R_1$ and $R_3 \to r_3 + 5R_1$.

$$\left[\begin{array}{ccccc} 1 & 2 & -3 & -1 & 2 \\ 0 & -1 & 12 & 5 & -9 \\ 0 & 1 & -7 & 0 & -6 \end{array}\right]$$

$R_3 \to r_3 + 1R_2$.

$$\begin{bmatrix} 1 & 2 & -3 & -1 & 2 \\ 0 & -1 & 12 & 5 & -9 \\ 0 & 0 & 5 & 5 & -15 \end{bmatrix}$$

$R_3 \to \frac{1}{5}R_3$.

$$\begin{bmatrix} 1 & 2 & -3 & -1 & 2 \\ 0 & -1 & 12 & 5 & -9 \\ 0 & 0 & 1 & 1 & -3 \end{bmatrix}$$

$R_1 \to r_1 + 3R_3$ and $R_2 \to r_2 + -12R_3$.

$$\begin{bmatrix} 1 & 2 & 0 & 2 & -7 \\ 0 & -1 & 0 & -7 & 27 \\ 0 & 0 & 1 & 1 & -3 \end{bmatrix}$$

$R_1 \to r_1 + 2R_2$.

$$\begin{bmatrix} 1 & 0 & 0 & -12 & 47 \\ 0 & -1 & 0 & -7 & 27 \\ 0 & 0 & 1 & 1 & -3 \end{bmatrix}$$

$R_2 \to -R_2$.

$$\begin{bmatrix} 1 & 0 & 0 & -12 & 47 \\ 0 & 1 & 0 & 7 & -27 \\ 0 & 0 & 1 & 1 & -3 \end{bmatrix}$$

So the solutions to the systems are $x_1 = -12, x_2 = 7, x_3 = 1$ and $x_1 = 47, x_2 = -27, x_3 = -3$.

Chapter 2.3

#4

$$\left[\begin{array}{ccccc|c} -1 & 5 & 2 & 3 & -1 & 6 \\ 3 & -15 & -6 & -8 & 5 & 2 \\ 5 & -25 & -10 & -13 & 9 & 4 \end{array}\right]$$

$R_2 \to r_2 + 3R_1$ and $R_3 \to r_3 + 1R_5$.

$$\left[\begin{array}{ccccc|c} -1 & 5 & 2 & 3 & -1 & 6 \\ 0 & 0 & 0 & 1 & 2 & 20 \\ 0 & 0 & 0 & 2 & 4 & 34 \end{array}\right]$$

$R_3 \to r_3 + -2R_2$.

$$\left[\begin{array}{ccccc|c} -1 & 5 & 2 & 3 & -1 & 6 \\ 0 & 0 & 0 & 1 & 2 & 20 \\ 0 & 0 & 0 & 0 & 0 & -6 \end{array}\right]$$

Because we have a row with 0's before the bar and a nonzero number after the bar, the system is inconsistent.

#6

$$\left[\begin{array}{cccc|c} 3 & -6 & 9 & -3 & 6 \\ 6 & -12 & 19 & -4 & 17 \\ 3 & -6 & 12 & 3 & 16 \\ 9 & -18 & 30 & -2 & 12 \end{array}\right]$$

Let's reduce this a little differently, to show the flexibility we have when putting things in row echelon form.

$R_2 \to r_2 + -2R_1$, $R_3 \to r_3 + -1R_1$, and $R_4 \to r_4 + -3R_1$.

$$\left[\begin{array}{cccc|c} 3 & -6 & 9 & -3 & 6 \\ 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & 3 & 6 & 10 \\ 0 & 0 & 3 & 7 & -6 \end{array}\right]$$

$R_3 \to r_3 + -3R_2$ and $R_4 \to r_4 + -3R_2$.

$$\left[\begin{array}{cccc|c} 3 & -6 & 9 & -3 & 6 \\ 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 1 & -21 \end{array}\right]$$

We can stop here; we already see a row which has zeros before the bar and a nonzero number after the bar. Even though the matrix is not yet in REF, continuing to put it in REF will not change the type of this row.

#10

Using the work from above, we can see that the rank of the coefficient matrix (the matrix to the left of the bar) is 3, since there are 3 rows with nonzero entries to the left of the bar. There are four rows with nonzero entries, overall, though. This indicates that the system is inconsistent, though we did not cover this material in class.

0.2 #18

We start by writing the corresponding matrix, then putting it into RREF.

$$\left[\begin{array}{ccc|c} 1 & -6 & 2 & 8 \\ 4 & -25 & 10 & 6 \\ 2 & -13 & 6 & k \end{array}\right]$$

$R_2 \to r_2 + -4R_1$ and $R_3 \to r_3 + -2R_1$.

$$\left[\begin{array}{ccc|l} 1 & -6 & 2 & 8 \\ 0 & -1 & 2 & -26 \\ 0 & -1 & 2 & k-16 \end{array}\right]$$

$R_3 \to r_3 + -1R_2$.

$$\left[\begin{array}{ccc|l} 1 & -6 & 2 & 8 \\ 0 & -1 & 2 & -26 \\ 0 & 0 & 0 & k-16+26 \end{array}\right]$$

In order to be consistent, we need $k - 16 + 26 = k - 10 = 0$, so $k = 10$.

0.3 8

#20

We again start by writing the corresponding equation.

$$\left[\begin{array}{ccc|c} 3 & -9 & 3 & a \\ 1 & -2 & -1 & b \\ 5 & -13 & 1 & c \end{array}\right]$$

$R_1 \leftrightarrow R_2$.

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & b \\ 3 & -9 & 3 & a \\ 5 & -13 & 1 & c \end{array}\right]$$

$R_2 \to r_2 + -3R_1$ and $R_3 \to r_3 + -5R_1$.

$$\left[\begin{array}{ccc|l} 1 & -2 & -1 & b \\ 0 & -3 & 6 & a-3b \\ 0 & -3 & 6 & c-5b \end{array}\right]$$

$R_3 \to r_3 + -1R_2$.

$$\left[\begin{array}{ccc|l} 1 & -2 & -1 & b \\ 0 & -3 & 6 & a-3b \\ 0 & 0 & 0 & c-5b-(a-3b) \end{array}\right]$$

To be consistent, we need $c - 5b - (a - 3b) = -a - 2b + c = 0$.

0.4 #22

We write the corresponding matrix.

$$\left[\begin{array}{ccc|l} 1 & -3 & 2 & -1 \\ -4 & 12 & -7 & 8 \end{array}\right]$$

$R_2 \to r_2 + 4R_1$.

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \rightarrow r_1 + -2R_2$.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

We now identify our basic and free variables. The leftmost ones occur in columns 1 and 3, so x_1 and x_3 are basic. That leaves x_2 free. Therefore, we will set $x_2 = t$.

Note that the second row gives $x_3 = 4$ immediately.

The first row tells us that $x_1 - 3x_2 = -9$. Substituting in for x_2, we see that $x_1 = 3t - 9$.

Therefore, the set of solutions to this system have the form $x_1 = 3t - 9, x_2 = t, x_3 = 4$, where t ranges over all real numbers.