

MATH 234 AL1 SECOND EXAM

THURSDAY, OCTOBER 18TH, 2007, 8AM

Name: \_\_\_\_\_ Section: \_\_\_\_\_

Show all work for full credit. No calculators are allowed.

Problem	Score
1	
2	
3	
4	
5	
6	
Total	of 100

You may need one or more of the following formulas:

Circle:  $A = \pi r^2$ ,  $C = 2\pi r$ ;

Cylinder:  $V = \pi r^2 h$ ,  $S_A = 2\pi r^2 + 2\pi r h$ ;

Sphere:  $V = \frac{4}{3}\pi r^3$ ,  $S_A = 4\pi r^2$ ;

Box:  $V = xyz$ ;  $S_A = 2xy + 2yz + 2zx$ .

**Problem 1** (15 pts). A spherical balloon is being filled with helium at a rate of  $5\text{in}^3/\text{second}$ . When the volume of the balloon is  $36\pi$  inches, at what rate is the radius increasing?

**Problem 2** (10 pts each). Let the cost and demand functions of a particular commodity be given by  $C(x) = 9x^2 + 4x + 81$  and  $p(x) = 5x + 36$ , respectively.

- (1) Find the production level  $x$  that maximizes profit. Justify.
- (2) Find the production level  $x$  that minimizes average cost.

**Problem 3** (10 pts each). Find  $\frac{dy}{dx}$

(1)  $y = \frac{1}{(x^2+1)^3}$

(2)  $x^3 = xy - y^2$ .

**Problem 4** (10 pts each). Consider the following function:

$$f(x) = 3x^5 - 40x^3.$$

- (1) Find the interval(s) where the function is concave up and the interval(s) where it is concave down.
- (2) Identify the inflection points. Justify.

**Problem 5** (15 pts). A closed cylindrical container is constructed with two types of cardboard. The first type is used for the top and bottom and costs 4 cents per  $cm^2$  and the second type, used for the sides, costs 2 cents per  $cm^2$ . If the container is to have volume  $32\pi$ , what dimensions minimize the construction cost?

**Problem 6** (10 pts). Find the absolute extrema of the function

$$f(x) = \frac{x^2}{x+2}$$

on the interval  $[-1, 1]$ .