Version A

Lecture 11

Solutions

1.

Determine intervals of concavity for the function

$$f(x) = 2x^6 - 20x^4 + 7x - 3$$

That is, determine where the graph of f is concave upward and where it is concave downward.

A. **Concave up for $x < -2$ and for $x > 2$ and concave down for $-2 < x < 0$ and for $0 < x < 2$.**

B. **Concave down for $x < -2$ and for $-2 < x < 0$ and concave up for $0 < x < 2$ and for $x > 2$.**

C. **Concave up for $x < -2$ and for $-2 < x < 0$ and concave down for $x > 2$ and for $0 < x < 2$.**

D. **Concave down for $x < -2$ and for $x > 2$ and concave up for $-2 < x < 0$ and for $0 < x < 2$.**

Solution:

We find that

$$f'(x) = 12x^5 - 80x^3 + 7$$

and

$$f''(x) = 60x^4 - 240x^2$$
$$= 60x^2(x^2 - 4)$$
$$= 60x^2(x - 2)(x + 2)$$

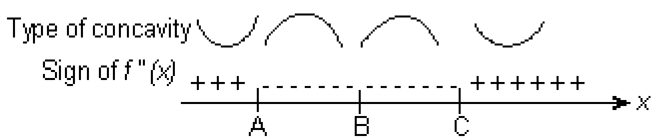
The second derivative $f''(x)$ is continuous for all x and $f''(x) = 0$ for $x = 0$, $x = 2$, and $x = -2$.

These numbers divide the x axis into four intervals on which $f''(x)$ does not change sign; namely, $x < -2$, $-2 < x < 0$, $0 < x < 2$, and $x > 2$.

Evaluating $f''(x)$ at test numbers in each of these intervals (say, at $x = -4$, $x = -1$, $x = 1$, and $x = 4$, respectively), we find that

$$f''(-4) = 11520 > 0$$
$$f''(-1) = -180 < 0$$
$$f''(1) = -180 < 0$$
$$f''(4) = 11520 > 0$$

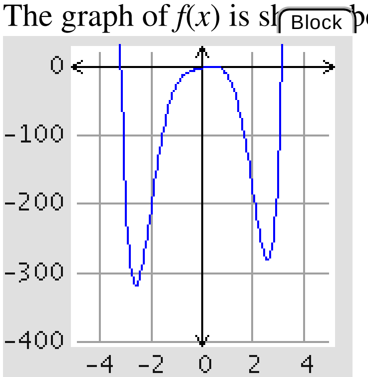
Thus, the graph of $f(x)$ is concave up for $x < -2$ and for $x > 2$ and concave down for $-2 < x < 0$ and for $0 < x < 2$, as indicated in this concavity diagram.



A = −2, B = 0, C = 2

Intervals of concavity for $f(x) = 2x^6 - 20x^4 + 7x - 3$

The graph of $f(x)$ is sh Block below.



The graph of $f(x) = 2x^6 - 20x^4 + 7x - 3$.

Correct answer is: A

2.

Determine where the graph of $f(x) = x^4 - 12x^3 + 3x - 5$ is concave upward and concave downward. Find the coordinates of all inflection points. Be sure to use commas to separate the coordinate of each point in your answer.

The inflection points are:(____),(____)

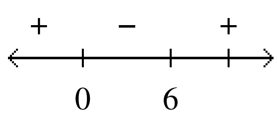
Solution:

$f(x) = x^4 - 12x^3 + 3x - 5$

$f'(x) = 4x^3 - 36x^2 + 3$

$f''(x) = 12x^2 - 72x = 0$ when $x = 0, 6$.

$f(0) = -5, f(6) = -1283$.



Thus $(0, -5)$, $(6, -1283)$ are inflection points.

Correct answer is: The inflection points are:(0, − 5),(6, − 1283)

3.

Find all inflection points of the given function.

$$g(x) = x^{7/9}$$

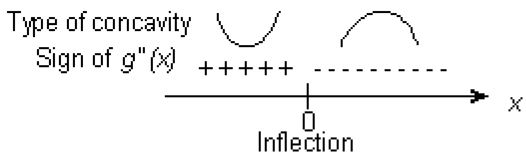
Inflection point: (____ , ____)

Solution:

The function $g(x)$ is continuous for all x and since

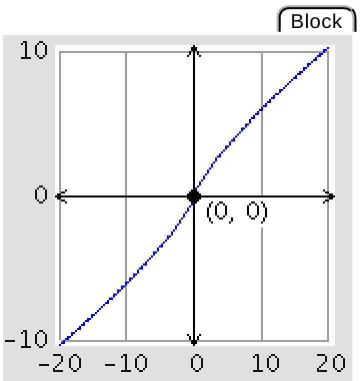
$$g'(x) = \frac{7}{9}x^{-2/9} \text{ and } g''(x) = \frac{-14}{81}x^{-11/9}$$

it follows that $g''(x)$ is never 0 but does not exist when $x = 0$. Testing the sign of $g''(x)$ on each side of $x = 0$, we obtain the results displayed in this concavity diagram:



Since the concavity of the graph changes at $x = 0$ and $g(0) = 0$, there is an inflection point at the origin, (0, 0).

The graph of g is shown in the figure below.



The graph of $g(x) = x^{7/9}$ has an inflection point at the origin (0, 0).

Correct answer is: Inflection point: (0, 0)

4.

Find the critical points of $f(x) = 2x^3 + 3x^2 - 120x - 10$ and use the second derivative test to classify each critical point as a relative maximum or minimum.

Rel. maximum: (____ , ____). Rel. minimum: (____ , ____).

Solution:

Since the first derivative

$$f'(x) = 6x^2 + 6x - 120 = 6(x + 5)(x - 4)$$

is zero when $x = -5$ and $x = 4$, the corresponding points $(-5, 415)$ and $(4, -314)$ are the critical points of f. To test these points, compute the second derivative

$$f''(x) = 12x + 6$$

and evaluate it at $x = -5$ and $x = 4$. Since

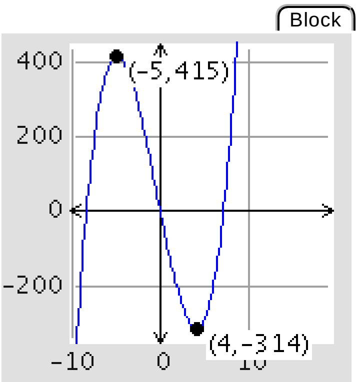
$$f''(-5) = -54 < 0$$

it follows that the critical point (−5, 415) is a relative maximum, and since

$$f''(4) = 54 > 0$$

it follows that the critical point (4, −314) is a relative minimum.

For reference, the graph of f is sketched below.



The graph of $f(x) = 2x^3 + 3x^2 - 120x - 10$.

Correct answer is: Rel. maximum: (− 5, 415). Rel. minimum: (4, − 314).

5.

Use the second derivative test to find the relative maxima and minima of $f(x) = 3x + 7 + \dfrac{48}{x}$.

Relative maximum: (____); Relative minimum: (____)

Solution:

$$f(x) = 3x + 7 + \frac{48}{x}$$

$$f'(x) = 3 - \frac{48}{x^2}$$

$$= \frac{3(x-4)(x+4)}{x^2}$$

$f'(x) = 0$ when $x = -4, 4$

$f''(x) = \frac{96}{x^3}$

$f''(-4) = -\frac{3}{2} < 0$ and $f''(4) = \frac{3}{2} > 0$;

So (−4, −17) is a relative maximum, (4, 31) is a relative minimum.

Correct answer is: Relative maximum: $(-4, -17)$; Relative minimum: $(4, 31)$

6.

Select the graph of a function that has all of the following properties:

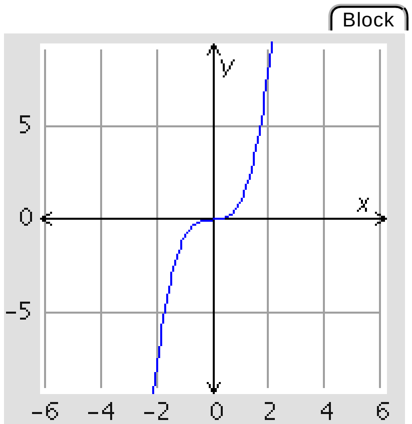
$f'(x) > 0$ for $-3 < x < 0$ and $3 < x < 6$

$f'(x) < 0$ for $-6 < x < -3$ and $0 < x < 3$

$f''(x) < 0$ for $-6 < x < -5$, $-1 < x < 1$, and $5 < x < 6$

$f''(x) > 0$ for $-5 < x < -1$ and $1 < x < 5$

Block

A. 

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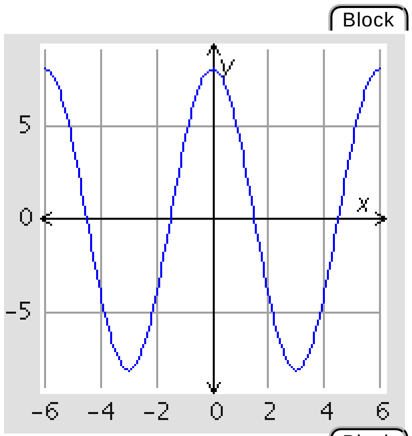
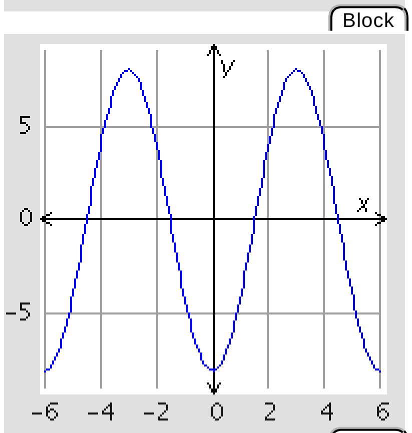


C.



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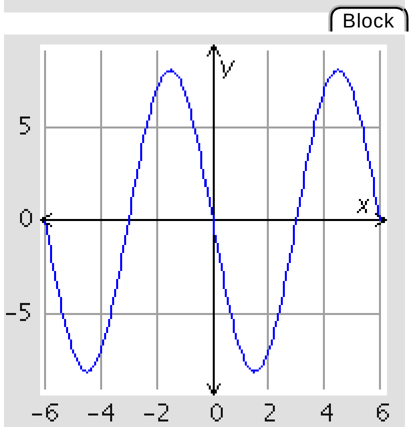


Solution:

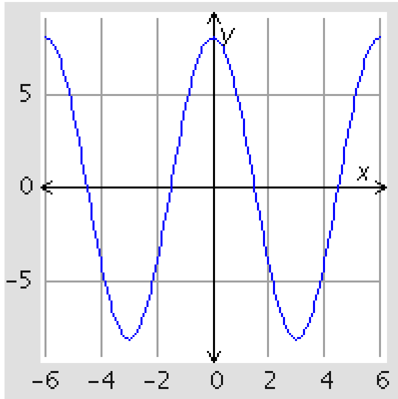
The curve rises for $-3 < x < 0$ and $3 < x < 6$.

It falls when $-6 < x < -3$ and $0 < x < 3$.

The curve is concave down for $-6 < x < -5$, $-1 < x < 1$, and $5 < x < 6$.

The curve is concave up for $-5 < x < -1$ and $1 < x < 5$.

Here is the possible graph.



Correct answer is: B

7.

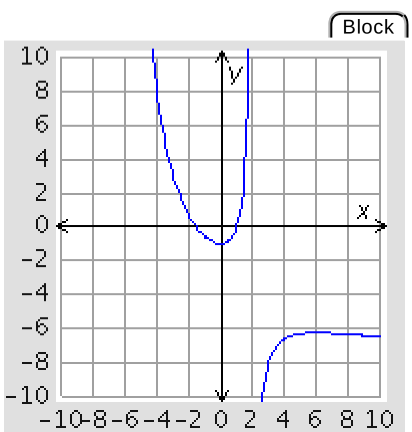
Select the graph of a function *f* that has all of the following properties:

(a) The graph has discontinuities at $x = 2$ and $x = -6$.

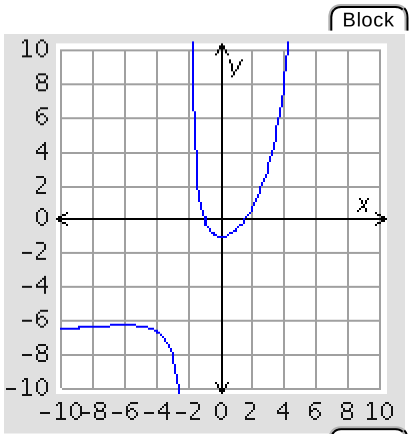
(b) $f'(x) > 0$ for $x < 0$

(c) $f'(x) < 0$ for $x > 0$

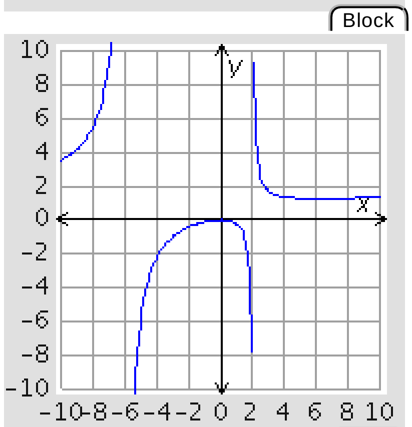
A.



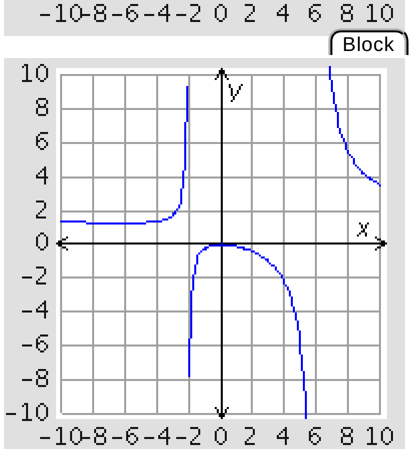
B.



C.



D.

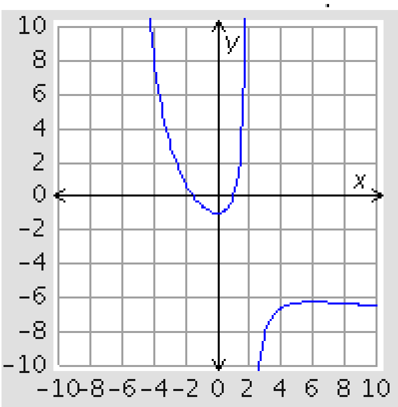


Solution:

(a) $x = 2$ and $x = -6$ denote discontinuities.

(b) $f\,'(x) > 0$ for $x < 0$.

(c) $f\,'(x) < 0$ for $x > 0$.



Correct answer is: A

