

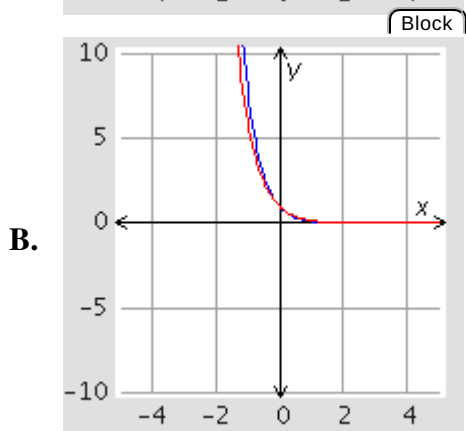
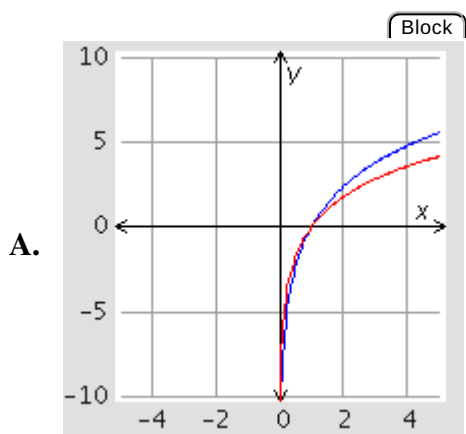
# Lecture 15 Solutions

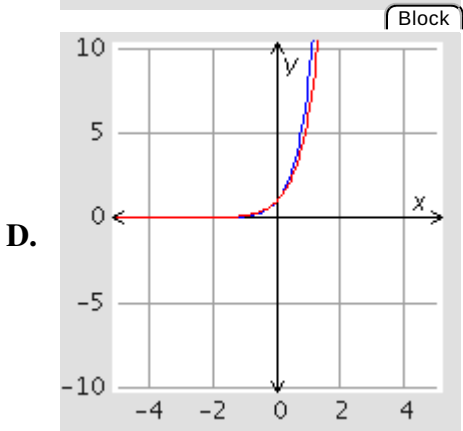
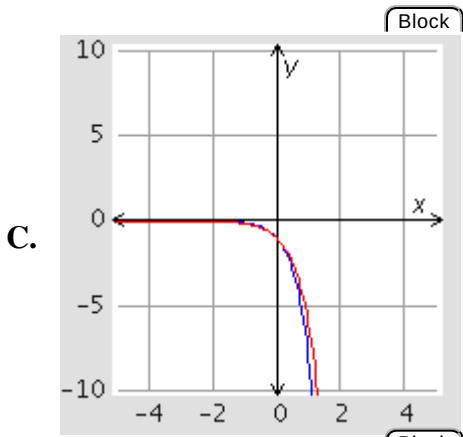
## Solutions

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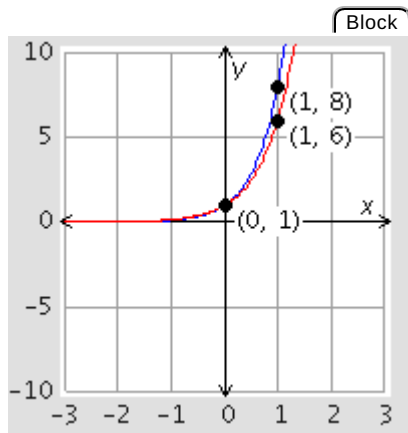
1.

Select the sketch of the curves  $y = 8^x$  and  $y = 6^x$ .





Solution:



Correct answer is: D

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2.

**Evaluate the exponential expression:**

$$5^4$$

$$5^8$$

Solution:

$$\frac{5^4}{5^8} = 5^{4-8} = 5^{-4} = \frac{1}{625}$$

Correct answer is:  $\frac{1}{625}$

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3.

**Evaluate the exponential expression:**

$$\left(\frac{3}{5}\right)^2$$

Solution:

$$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{9}{25}$$

Correct answer is:  $\frac{9}{25}$

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4.

**If  $f(x) = 2^{x^2 - x}$ , find all values of  $x$  such that  $f(x) = 64$ .**

$$x = \underline{\quad} \text{ or } x = \underline{\quad}$$

Solution:

The equation  $f(x) = 64 = 2^6$  is satisfied if and only if

$$2^{x^2 - x} = 2^6$$

$$x^2 - x = 6 \quad \text{since } b^x = b^y \text{ only when } x = y$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0 \quad \text{factor}$$

$$x = 3, x = -2$$

Thus,  $f(x) = 64$  if and only if  $x = 3$  or  $x = -2$ .

Correct answer is:  $x = 3$  or  $x = -2$

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5.

**Find all real numbers  $x$  that satisfy the equation  $5^{15-x} = 25^x$ .**

$$x = \underline{\quad}$$

Solution:

$$5^{15-x} = 25^x$$

$$5^{15-x} = (5^2)^x$$

$$5^{15-x} = 5^{2x}$$

By the equality rule of exponential functions,

$$15 - x = 2x, \text{ or } x = 5.$$

Correct answer is: 5

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6.

**Suppose \$700 is invested at an annual interest rate of 8%. Compute the balance after 8 years if the interest is compounded monthly. Round your final answer to two decimal places.**

**Your Answer: \$ \_\_\_\_\_**

Solution:

Take  $t = 8$ ,  $P = 700$ ,  $r = 0.08$ , and  $k = 12$  to get

$$B(8) = 700 \left( 1 + \frac{0.08}{12} \right)^{96} \approx \$1324.72$$

Correct answer is: 1324.72

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7.

**Suppose \$1200 is invested at an annual interest rate of 6%. Compute the balance after 11 years if the interest is compounded annually. Round your final answer to the nearest cent.**

**Your Answer: \$ \_\_\_\_\_**

Solution:

The balance after  $t$  years, if  $P$  dollars is invested at an annual interest rate  $r$  and interest is compounded  $k$  times per year, will be,

$$B(t) = P \left( 1 + \frac{r}{k} \right)^{kt} \text{ dollars.}$$

When  $t = 11$ ,  $P = 1200$ ,  $r = 0.06$ , and  $k = 1$ ,

$$B(11) = 1200 \left( 1 + 0.06 \right)^{11} \approx \$2277.96$$

Correct answer is: 2277.96

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8.

**Suppose \$3500 is invested at an annual interest rate of 6%. Compute the balance after 12 years if the interest is compounded continuously. Round your final answer to the nearest cent.**

**Your Answer: \$ \_\_\_\_\_**

Solution:

With semi-annual compounding,

$$B(t) = Pe^{rt}.$$

$$\begin{aligned} B(t) &= (3500)e^{(0.06)(12)} \\ &= (3500)e^{0.72} \\ &= \$7190.52 \end{aligned}$$

Correct answer is: 7190.52

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9.

**Sue is about to enter college. When she graduates 4 years from now, she wants to take a trip to Europe that she estimates will cost \$8000. How much should she invest now at 5% to have enough for the trip if interest is compounded continuously?**

**Round your answer to the nearest cent.**

**She has to invest \$ \_\_\_\_\_ .**

Solution:

The required future value is  $F = \$8000$  in  $t = 4$  years with  $r = 0.05$ .

For continuous compounding, the present value is

$$P = 8000e^{-0.05(4)} = \$6549.85$$

Correct answer is: 6549.85

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10.

**How much money should be invested today at 8% compounded continuously so that it will be worth \$16000 in 25 years? Round your answer to the nearest cent.**

**Your Answer: \$ \_\_\_\_\_**

Solution:

$B(t) = (16000)$ ,  $r = 0.08$ , and  $t = 25$ .

$$B(t) = Pe^{rt}, \text{ so } P = \frac{16000}{e^{0.08 \cdot 25}} \approx \$2165.36$$

Correct answer is: 2165.36

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