

lecture 16

Solutions

1.

Evaluate

$$\log_4 65536 = \underline{\hspace{2cm}}$$

Solution:

$$\log_4 65536 = 8 \text{ since } 4^8 = 65536.$$

Correct answer is: 8

2.

Solve the following equation for x :

$$\log_x 1296 = 4$$

$$x = \underline{\hspace{2cm}}$$

Solution:

$$\log_x 1296 = 4 \text{ means}$$

$$x^4 = 1296$$

$$x = (1296)^{1/4} = 6$$

Correct answer is: 6

3.

Find

$$\ln e = \underline{\hspace{2cm}}$$

Solution:

According to the definition, $\ln e$ is the unique number c such that $e = e^c$. Clearly this number is $c = 1$. Hence, $\ln e = 1$.

Correct answer is: 1

4.

Simplify the following expression:

$$\ln e^{9x}$$

Solution:

$\ln e^{9x}$ is the unique number b for which $e^{9x} = e^b$. Clearly, this number b is $9x$. Hence, $\ln e^{9x} = 9x$.

Correct answer is: $9x$

5.

How quickly will money double if it is invested at an annual interest rate of 6.2% compounded continuously? Round your answer to the nearest month.

_____ years and _____ months.

Solution:

$$B(t) = Pe^{0.062t}$$

$$2 = e^{0.062t} \text{ and } t = \frac{\ln 2}{0.062} \approx 11.1798 \text{ years}$$

or 11 years 2 months.

Correct answer is: 11 years and 2 months.

6.

Differentiate the function $f(x) = x^7 \ln x$.

$$f'(x) = \underline{\hspace{2cm}}$$

Solution:

Combine the product rule with the formula for the derivative of $\ln x$ to get

$$f'(x) = x^7 \left(\frac{1}{x} \right) + 7x^6 \ln x = x^6 + 7x^6 \ln x$$

Correct answer is: $f'(x) = x^6 + 7x^6 \ln x$

7.

Differentiate the function $f(x) = \ln(4x^5 + 5)$.

A. $\frac{20x^4}{4x^5}$

B. $\frac{4x^4}{4x^5 + 5}$

C. $\frac{20x^4}{x^5 + 5}$

D. $\frac{20x^4}{4x^5 + 5}$

Solution:

Here, we have $f(x) = \ln u$, where $u(x) = 4x^5 + 5$. Thus,

$$\begin{aligned} f'(x) &= \frac{1}{u} \frac{du}{dx} = \frac{1}{4x^5 + 5} \frac{d}{dx} (4x^5 + 5) \\ &= \frac{4(5x^4)}{4x^5 + 5} \\ &= \frac{20x^4}{4x^5 + 5} \end{aligned}$$

Correct answer is: D

8.

Differentiate the given function.

$$f(x) = e^{11x}$$

$$f'(x) = \underline{\hspace{2cm}}$$

Solution:

$$\begin{aligned} f'(x) &= e^{11x} \frac{d}{dx} (11x) \\ &= 11e^{11x} \end{aligned}$$

Correct answer is: $f'(x) = 11e^{11x}$

9.

Differentiate the given function.

$$f(x) = \frac{4e^{6x}}{x}$$

$$f'(x) = \underline{\hspace{2cm}}$$

Solution:

$$\begin{aligned} f'(x) &= \frac{24xe^{6x} - 4e^{6x}(1)}{x^2} \\ &= \frac{4e^{6x}[6x - 1]}{x^2} \end{aligned}$$

Correct answer is: $f'(x) = \frac{4e^{6x}(6x - 1)}{x^2}$
