Version A

Lecture 18

Solutions

1.

The rate at which a postal clerk can sort mail is a function of the clerk's experience. Suppose the postmaster of a large city estimates that after t months on the job, the average clerk can sort $Q(t) = 680 - 410e^{-0.4t}$ letters per hour. How many letters can a new employee sort per hour?

The number of letters a new employee can sort per hour is ____ .

Solution:

The number of letters a new employee can sort per hour is

$$Q(0) = 680 - 410e^{0} = 270$$

Correct answer is: 270

2.

The rate at which a postal clerk can sort mail is a function of the clerk's experience. Suppose the postmaster of a large city estimates that after t months on the job, the average clerk can sort $Q(t) = 800 - 300e^{-0.5t}$ letters per hour. How many letters can a clerk with 4 months' experience sort per hour? Round your answer to the nearest letter.

Your Answer: ____ letters per hour

Solution:

After 4 months, the average clerk can sort

$Q(4) = 800 - 300e^{-0.5(4)} = 800 - 300e^{-2} \approx 759$ letters per hour

Correct answer is: 759

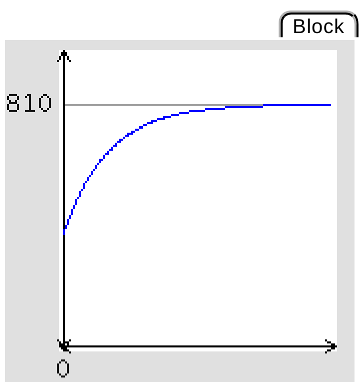
3.

The rate at which a postal clerk can sort mail is a function of the clerk's experience. Suppose the postmaster of a large city estimates that after t months on the job, the average clerk can sort $Q(t) = 810 - 430e^{-0.6t}$ letters per hour. Approximately how many letters will the average clerk ultimately be able to sort per hour?

The average clerk will ultimately be able to sort approximately ____ letters per hour.

Solution:

As t increases without bound, Q(t) approaches 810. Hence, the average clerk will ultimately be able to sort approximately 810 letters per hour. The graph of the function $Q(t)$ is sketched in figure blow.



Worker efficiency $Q(t) = 810 - 430e^{-0.6t}$.

Correct answer is: 810

4.

A drink is taken outside on a cold winter day when the air temperature is -6^{o}C. According to a principle of physics called Newton's law of cooling, the temperature *T* (in degrees Celsius) of the drink *t* minutes after being taken outside is given by a function of the form

$$T(t) = -6 + Ae^{-kt}$$

where *A* and *k* are constants. Suppose the temperature of the drink is 28^{o}C when it is taken outside and 20 minutes later, is 6^{o}C. Use this information to determine *A* and *k*.

$A =$ ____ , $k =$ ____

Solution:

Since the temperature is 28^{o}C when $t = 0$, we have

$$T_0 = T(0) = -6 + Ae^{-k(0)} = 28$$

$$-6 + A = 28$$

$$A = 34$$

And since the temperature is 6^{o}C when $t = 20$, we have

$$T(20) = -6 + 34e^{-k(20)} = 6$$

$$34e^{-k(20)} = 6 + 6 = 12$$

$$e^{-k(20)} = \frac{12}{34} = \frac{6}{17}$$

$$\ln(e^{-k(20)}) = \ln\left(\frac{6}{17}\right) \quad \textit{take logarithms on both sides}$$

$$-k(20) = \ln\left(\frac{6}{17}\right) \quad \textit{use } \ln(e^x) = x$$

$$k = -\frac{1}{20}\ln\frac{6}{17} = \frac{1}{20}\ln\frac{17}{6}$$

Correct answer is: $A = 85,\ k = \frac{1}{20}\ln\frac{17}{6}$

5.

Public health records indicate that *t* weeks after the outbreak of a certain form of influenza, approximately $Q(t) = \dfrac{21}{1 + 14e^{-1.7t}}$ **thousand people had caught the disease. How many had caught the disease by the end of the 9 weeks? Round your intermediate calculations to three decimal places and your final answer to the nearest whole number.**

About ____ had contracted the disease by the ninth week.

Solution:

$Q(9) = \dfrac{21}{1 + 14(0)}$, so 21000 people have caught the disease.

Correct answer is: 21000

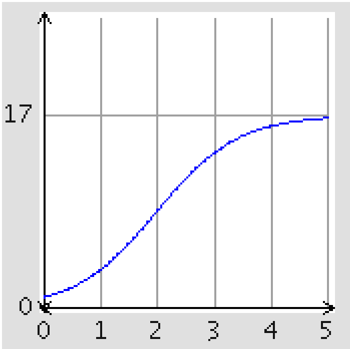
6.

Public health records indicate that *t* weeks after the outbreak of a certain form of influenza, approximately $Q(t) = \dfrac{17}{1 + 16e^{-1.4t}}$ **thousand people had caught the disease. If the trend continues, approximately how many people will eventually contract the disease?**

Approximately ____ people will eventually contract the disease.

Solution:

Since $Q(t)$ approaches 17 as t increases without bound, it follows that approximately 17000 people will eventually contract the disease. For reference, the graph is sketched in the figure below.



The spread of an epidemic $Q(t) = \dfrac{17}{1 + 16e^{-1.4t}}$.

Correct answer is: 17000

7.

The economics editor at a major publishing house estimates that if *x* thousand complimentary copies are distributed to professors, the first-year sales of a certain new text will be $f(x) = 15 - 10e^{-0.5x}$ thousand copies. Currently, the editor is planning to distribute 7000 complimentary copies. Use marginal analysis to estimate the increase in first-year sales that will result if 2000 additional complimentary copies are distributed. Round all intermediate calculations to the third decimal place and round your final answer to the nearest whole number.

Approximately ____ additional copies will be sold.

Solution:

$\Delta f \approx f'(x)$, where x is the current number of complimentary copies. $f'(x) = 5e^{-0.5x}$.

$\Delta f \approx f'(7) = 5e^{-0.5(7)} \approx 0.151$.

So, approximately 151 additional copies will be sold.

Correct answer is: -3.5

8.

The ratio of ^{14}C to ^{12}C in a sample (e.g., a fossil or an artifact) is given by a function of the form $R(t) = R_0e^{-kt}$. The half-life of ^{14}C is 5730 years. By comparing $R(t)$ to R_0, archaeologists can estimate the age of the sample.

An archaeologist has found a fossil in which the ratio of ^{14}C to ^{12}C is $\frac{1}{8}$ the ratio found in the atmosphere. Approximately how old is the fossil? Round your intermediate results to six decimal places and your final answer to the nearest hundred.

The fossil is approximately ____ years old.

Solution:

The age of the fossil is the value of t for which $R(t) = \frac{1}{8}R_0$; that is, for which

$$\frac{1}{8}R_0 = R_0e^{-kt}$$

Dividing by R_0 and taking logarithms, you find that

$$\frac{1}{8} = e^{-kt}$$

$$\ln\frac{1}{8} = -kt$$

and

$$t = \frac{-\ln\frac{1}{8}}{k} = \frac{\ln 8}{k}$$

The half-life h satisfies $h = \frac{\ln 2}{k}$, and since ^{14}C has half-life $h = 5730$ years, you have

$$k = \frac{\ln 2}{h} = \frac{\ln 2}{5730} \approx 0.000121$$

Therefore, the age of the fossil is

$$t = \frac{\ln 8}{k} = \frac{\ln 8}{0.000121} \approx 17200$$

That is, the fossil is approximately 17200 years old.

Correct answer is: 17200
