Version A

Lecture 20 Solutions

Solutions

1.

Specify the substitution you would choose to find the integral $\int (3x + 5)^{9/4}\, dx$.

Substitution u = ____

Solution:

We choose $u = 3x + 5$ and obtain

$$du = 3\, dx \quad \text{so that} \quad dx = \frac{1}{3}\, du$$

Then the integral becomes

$$\int (3x + 5)^{9/4}\, dx = \int (u)^{9/4} \left(\frac{1}{3}\, du \right)$$

Correct answer is: Substitution u = 3x + 5

2.

Specify the substitution you would choose to find the integral $\int t(2 + t^2)^5\, dt$.

Substitution u = ____

Solution:

We choose $u = 2 + t^2$ and obtain

$$du = 2t\,dt \quad \text{so that} \quad dt = \frac{1}{2t}\,du$$

Then the integral becomes

$$\int t(2 + t^2)^5\,dt = \int t(u)^5\left(\frac{1}{2t}\,du\right)$$

Correct answer is: Substitution $u = 2 + t^2$

3.

Find $\int \sqrt{10x + 19}\;dx$.

A. $\frac{1}{15}(10x + 19)^{3/2} + C$

B. $\frac{2}{3}(10x + 19)^{3/2} + C$

C. $5(10x + 19)^{-1/2} + C$

D. $5(10x + 19)^{3/2} + C$

Solution:

We choose $u = 10x + 19$ and obtain

$$du = 10\,dx \quad \text{so that} \quad dx = \frac{1}{10}\,du$$

Then the integral becomes

$$\int \sqrt{10x + 19}\;dx = \int \sqrt{u}\left(\frac{1}{10}\,du\right)$$

$$= \frac{1}{10}\int u^{1/2}du \qquad \textit{since } \sqrt{u} = u^{1/2}$$

$$= \frac{1}{10}\frac{u^{3/2}}{3/2} + C = \frac{1}{15}u^{3/2} + C \qquad \textit{power rule}$$

$$= \frac{1}{15}(10x + 19)^{3/2} + C \qquad \textit{substitute } 10x + 19 \textit{ for } u$$

Correct answer is: A

4.

Find $\int \frac{6x}{x-8}\,dx$.

A. $\int \frac{6x}{x-8}\,dx = 6x - 48 + \ln|x - 8| + C$

B. $\int \frac{6x}{x-8}\,dx = 6x - 6 + 48\ln|x - 8| + C$

C. $\int \frac{6x}{x-8}\,dx = 6x - 48 + 48\ln|x - 8| + C$

D. $\int \frac{6x}{x-8}\,dx = 6x - 48 - 48\ln|x - 8| + C$

Solution:

Following our guidelines, we substitute for the denominator of the integrand, so that $u = x - 8$ and $du = dx$. Since $u = x - 8$, we also have $x = u + 8$. Thus,

$$\int \frac{6x}{x-8}\,dx = \int \frac{6(u+8)}{u}\,dx$$

$= 6\int \left(1 + \frac{8}{u}\right) du$ *divide*

$= 6u + 48 \ln |u| + C$ *constant and logarithmic rules*

$= 6x - 48 + 48 \ln |x - 8| + C$ *substitute x − 8 for u*

Correct answer is: C

5.

Find $\int \frac{6(\ln x)^6}{x}\, dx$**.**

____ $+ C$

Solution:

Because

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

the integrand

$$\frac{6(\ln x)^6}{x} = 6(\ln x)^6 \left(\frac{1}{x}\right)$$

is a product in which one factor $\frac{1}{x}$ is the derivative of an expression $\ln x$ that appears in the other factor. This suggests that you let $u = \ln x$ with $du = \frac{1}{x}\, dx$. Substituting $u = \ln x$ and $du = \frac{1}{x}\, dx$, you get

$$\int \frac{6(\ln x)^6}{x}\, dx = \int 6(\ln x)^6 \left(\frac{1}{x}\, dx\right) dx$$

$$= 6\int u^6\, du = \frac{6}{7} u^7 + C$$

$$= \frac{6}{7} (\ln x)^7 + C$$ *substitute ln x for u*

Correct answer is: $\frac{6}{7}(\ln x)^7 + C$

6.

Find $\int \frac{1}{4 + e^{-x}}\, dx$.

A. $\int \frac{1}{4 + e^{-x}}\, dx = \ln |e^x + 4| + C$

B. $\int \frac{1}{4 + e^{-x}}\, dx = \frac{1}{4} \ln |4e^x + 1| + C$

C. $\int \frac{1}{4 + e^{-x}}\, dx = \frac{1}{4} \ln |4e^{-x} + 1| + C$

D. $\int \frac{1}{4 + e^{-x}}\, dx = \frac{1}{4} \ln |4e^x - 1| + C$

Solution:

You may try to substitute $w = 4 + e^{-x}$. However, this is a dead end because $dw = -e^{-x}\, dx$ but there is no e^{-x} term in the numerator of the integrand. Instead, note that

$$\frac{1}{4 + e^{-x}} = \frac{1}{4 + \frac{1}{e^x}} = \frac{1}{\frac{4e^x + 1}{e^x}}$$

$$= \frac{e^x}{4e^x + 1}$$

Now, if you substitute $u = 4e^x + 1$ with $du = 4e^x\,dx$ into the given integral, you get

$$\int \frac{1}{4 + e^{-x}}\,dx = \int \frac{e^x}{4e^x + 1}\,dx = \frac{1}{4}\int \frac{1}{4e^x + 1}(4e^x\,dx)$$

$$= \frac{1}{4}\int \frac{1}{u}\,du$$

$$= \frac{1}{4}\ln|u| + C$$

$$= \frac{1}{4}\ln|4e^x + 1| + C \qquad \textit{substitute } 4e^x + 1 \textit{ for } u$$

Correct answer is: B

7.

Find the indicated integral.

$$\int \frac{5y^7}{y^8 + 4}\,dy$$

A. $\frac{5}{7}\ln|y^8 + 4| + C$

B. $\frac{5}{8}\ln|y^8 + 4| + C$

C. $\frac{5}{8}\ln|y^8| + 4 + C$

D. $\frac{5}{8}\ln|y^7 + 8| + C$

Solution:

Let $u = y^8 + 4$. Then $\frac{du}{dy} = 8y^7$, or $\frac{1}{8}\,du = y^7\,dy$.

Then the integral becomes

$$\int \frac{5y^7}{y^8+4}\,dy = 5\int \frac{1}{y^8+4}\,y^7dy = 5\int \frac{1}{u}\cdot\frac{1}{8}\,du$$

$$= \frac{5}{8}\int \frac{1}{u}\,du$$

$$= \frac{5}{8}\ln|y^8+4| + C$$

Correct answer is: B

8.

Find the indicated integral.

$$\boldsymbol{I = \int \frac{17e^{\sqrt{9x}}}{\sqrt{9x}}\,dx}$$

$I =$ ____ $+\ C$

Solution:

Let $u = \sqrt{9x}$ then $du = \frac{9}{2\sqrt{9x}}\,dx$ and

$$I = \int \frac{17e^{\sqrt{9x}}}{\sqrt{9x}}\,dx$$

$$= \frac{34}{9}\int e^{\sqrt{9x}}\,\frac{9}{2\sqrt{9x}}\,dx$$

$$= \frac{34}{9} \int e^u \, du$$

$$= \frac{34}{9} e^u + C$$

$$= \frac{34}{9} e^{\sqrt{9x}} + C$$

Correct answer is: $I = \frac{34}{9} e^{\sqrt{9x}} + C$

9.

A tree has been transplanted and after x years is growing at the rate of

$1 + \frac{1}{(x+1)^2}$ meters per year. After 3 years it has reached a height of

9 meters. How tall was it when it was transplanted?

The plant was ____ meters tall when the plant was transplanted.

Solution:

The height of the plant when it was transplanted $h(x)$ is found by integrating $h'(x)$ with respect to x. We can integrate 1 directly. For the term $1/(x + 1)^2$, We can use the substitution

$$u = x + 1, \qquad du = dx$$

to get

$$\int 1 + \frac{1}{(x+1)^2} \, dx = \int 1 \, dx + \int \frac{1}{(x+1)^2} \, dx$$

$$= \int 1 \, dx + \int \frac{1}{u^2} \, du$$

$$= x - \frac{1}{u} + C$$

Substitute $x + 1$ for u.

$$= x - \frac{1}{x+1} + C$$

Since $h = 9$ meters when $x = 3$ years, you find that

$$9 = 3 - \frac{1}{3+1} + C$$

$$C = 9 - 3 + \frac{1}{3+1} = \frac{25}{4}$$

so

$$h(x) = x - \frac{1}{x+1} + \frac{25}{4}$$

The tree was transplanted when $x = 0$ years and the corresponding height was

$$h(0) = 0 - 1 + \frac{25}{4}$$

$$= \frac{21}{4} = 5.25 \text{ meters}$$

The plant was 5.25 meters tall when the plant was transplanted.

Correct answer is: 5.25

10.

Find the indicated integral and check your answer by differentiation.

$$I = \int \frac{\ln 10x}{3x}\, dx$$

$I =$ ____ $+ C$

Solution:

Let $u = \ln 10x$. Then $\frac{du}{dx} = \frac{1}{10x} \cdot 10 = \frac{1}{x}$, or $du = \frac{1}{x}\, dx$.

$$\int \frac{\ln 10x}{3x}\, dx = \frac{1}{3} \int \ln 10x \cdot \frac{1}{x}\, dx$$

$$= \frac{1}{3}\int u\,du$$

$$= \frac{(\ln 10x)^2}{6} + C.$$

$$I' = \frac{\ln 10x}{3x}$$

Correct answer is: $I = \dfrac{(\ln 10x)^2}{6} + C$
