Version A

Lecture 24 Solutions

Solutions

1.

Evaluate the improper integral

$$\int_2^{+\infty} \frac{1}{3x^2}\,dx = \_\_\_\_$$

Solution:

First compute the integral from 2 to N and then let N approach infinity. Arrange your work as follows:

$$\begin{aligned}\int_2^{+\infty} \frac{1}{3x^2}\,dx &= \lim_{N\to+\infty} \int_2^{N} \frac{1}{3x^2}\,dx \\ &= \lim_{N\to+\infty} \left(-\frac{1}{3x}\Big|_2^N\right) \\ &= \lim_{N\to+\infty} \left(-\frac{1}{3N} + \frac{1}{6}\right) = \frac{1}{6}\end{aligned}$$

Correct answer is: $\int_2^{+\infty} \frac{1}{3x^2}\,dx = \frac{1}{6}$

2.

Evaluate the improper integral.

$$\int_6^{+\infty} \frac{17}{12x-12}\,dx$$

Solution:

$$\int_6^{\infty} \frac{17}{12x-12}\,dx = \lim_{N\to\infty} \int_6^{N} \frac{17}{12x-12}\,dx$$

$$= \lim_{N\to\infty} \frac{17}{12} \ln|12x-12|\Big|_6^N$$

$$= \frac{17}{12} \lim_{N\to\infty} \ln|12x-12|\Big|_6^N$$

$$= \frac{17}{12} \lim_{N\to\infty} \left(\ln(12N-12) - \ln(60)\right)$$

$$= \infty$$

So, the integral diverges.

Correct answer is: ∞

3.

Evaluate the improper integral.

$$\int_8^{+\infty} 9e^{-x}dx$$

Solution:

$$I = \int_8^{+\infty} 9e^{-x}dx = \lim_{N\to+\infty} \int_8^{N} 9e^{-x}dx$$

$$= -9 \lim_{N\to+\infty} e^{-x}\Big|_8^N$$

$$= 9e^{-8}$$

Correct answer is: $9e^{-8}$

4.

$f(x)$ is a probability density function for a particular random variable X. Use integration to find $P(0 \le X \le 6)$.

$$f(x) = \begin{cases} \frac{1}{24}(7 - x) & \text{if } 0 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

$P(0 \le X \le 6)$ = ____

Solution:

$$P(0 \le X \le 6) = \int_0^6 \frac{1}{24}(7 - x)\,dx = \frac{1}{24}\left(7x - \frac{x^2}{2}\right)\Big|_0^6$$

$$= \frac{1}{24}\left(42 - \frac{36}{2}\right) = 1$$

Correct answer is: 1

5.

The time interval between the arrivals of successive planes at a certain airport is measured by a random variable X with probability density function

$$f(x) = \begin{cases} 0.8e^{-0.8x} & \text{if } x \ge 0 \\ 0 & \text{if } x < 0 \end{cases}$$

where x is the time (in minutes) between the arrivals of a randomly selected pair of successive planes. What is the probability that two successive planes selected at random will arrive within 2 minutes of one another?

Solution:

$$P(X<2) = \int_0^2 0.8e^{-0.8x}dx$$
$$= -e^{-0.8x}\Big|_0^2$$
$$= -e^{-1.6} + 1$$

Correct answer is: $\frac{-1}{e^{1.6}} + 1$

6.

The probability density function for a continuous random variable X is given. Find the expected value $E(X)$.

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 5 \le x \le 7 \\ 0 & \text{otherwise} \end{cases}$$

$E(X) =$ ____

Solution:

The expected value of X is

$$E(X) = \int_{-\infty}^{\infty} xf(x)\,dx = \int_5^7 \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_5^7 = \frac{49}{4} - \frac{25}{4} = 6$$

Correct answer is: $E(X) = 6$
