Version A

Lecture 26 Solutions

Solutions

1.

Suppose $f(x, y) = (x - 1)^2 + 3xy^3$. Compute $f(2, -5)$.

$f(2, -5)$ = ____

Solution:

$$\begin{aligned} f(2, -5) &= (2 - 1)^2 + 3(2)(-5)^3 \\ &= 1 - 750 \\ &= -749. \end{aligned}$$

Correct answer is: -749

2.

Describe the domain of the given function.

$$f(x, y) = \frac{x + 8y}{5x + 7y}$$

A. All ordered pairs (x, y) of real numbers for which $y = \frac{5}{7}x$

B. All ordered pairs (x, y) of real numbers for which $y = -5x$

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C. **All ordered pairs (x, y) of real numbers for which $y \neq \frac{1}{8}x$**

D. **All ordered pairs (x, y) of real numbers for which $y \neq -\frac{5}{7}x$**

Solution:

Since division by any real number except zero is possible, the expression $f(x, y)$ can be evaluated for all ordered pairs (x, y) with $5x + 7y \neq 0$ or $y \neq \frac{-5}{7}x$.

Geometrically, this is the set of all points in the xy−plan except those on the line $y = -\frac{5}{7}x$.

Correct answer is: D

3.

Find the partial derivative f_x if $f(x, y) = 4x^5 + 2xy^4 + \frac{y}{4x}$.

$f_x(x, y) =$ ____

Solution:

To simplify the computation, begin by rewriting the function as

$$f(x, y) = 4x^5 + 2xy^4 + \frac{1}{4}yx^{-1}$$

To compute f_x, think of f as a function of x and differentiate the sum term by term, treating y as a constant to get

$$f_x(x, y) = 4(5)x^4 + 2(1)y^4 + \frac{1}{4}y(-x^{-2})$$

$$= 20x^4 + 2y^4 - \frac{y}{4x^2}$$

Correct answer is: $f_x(x, y) = 20x^4 + 2y^4 - \frac{y}{4x^2}$

4.

Find the partial derivative $\frac{\partial z}{\partial y}$ if $z = (x^2 + xy + y)^7$.

A. $7(x^2 + xy + x)^6(2x + y)$

B. $7(x^2 + xy + x)^6(x + 1)$

C. $7(x^2 + xy + x)^6$

D. $(x^2 + xy + x)^7(x + 1)$

Solution:

Holding x fixed and using the chain rule to differentiate z with respect to y, you get

$$\frac{\partial z}{\partial y} = 7(x^2 + xy + x)^6 \frac{\partial}{\partial y}(x^2 + xy + y)$$

$$= 7(x^2 + xy + x)^6(x + 1)$$

Correct answer is: B

5.

Compute all first-order partial derivative of the given function.

$$\boldsymbol{f(s, t) = \frac{11t}{6s}}$$

$f_s =$ ____ , $f_t =$ ____

Solution:

$$f(s, t) = \frac{11t}{6s} = \frac{11}{6} s^{-1} t$$

$$f_s = \frac{11}{6}(-1)s^{-2}t$$

$$= -\frac{11t}{6s^2}$$

$$f_t = \frac{11}{6} s^{-1} = \frac{11}{6s}$$

Correct answer is: $f_s = -\frac{11t}{6s^2}$, $f_t = \frac{11}{6s}$

6.

Compute the second-order partial derivative f_{yx} of the function

$$\boldsymbol{f(x, y) = xy^5 + 5xy^2 + 4x + 8}$$

A. $5y^4 + 10y$

B. 0

C. $5y^4 + 10$

D. $20xy^3 + 10x$

Solution:

Since

$$f_y = 5xy^4 + 10xy$$

it follows that

$$f_{yx} = 5y^4 + 10y$$

Correct answer is: A

7.

Find the second partial derivative f_{yy} if $f(x, y) = 2x^2y^3 + 6xy$.

f_{yy} = ____

Solution:

Since

$$f_y = 6x^2y^2 + 6x$$

we have

$$f_{yy} = 12x^2y$$

Correct answer is: $f_{yy} = 12x^2y$

8.

Find the second partial derivative f_{xy} if $f(x, y) = 8x^4y^4 + 10xy$.

f_{xy} = ____

Solution:

Since

$$f_x = 32x^3y^4 + 10y$$

if follows that

$f_{xy} = 128x^3y^3 + 10$

$= 2(64x^3y^3 + 5)$

Correct answer is: $f_{xy} = 2(64x^3y^3 + 5)$

9.

Find the second partial derivative f_{xx} if $f(x, y) = 3x^4y + 9xy$.

$f_{xx}=$ ____

Solution:

Since

$f_x = 12x^3y + 9y$

if follows that

$f_{xx} = 36x^2y$

Correct answer is: $f_{xx}= 36x^2y$

10.

Find the second partial.

$f(s, t) = 18\sqrt{8s^2 + 5t^2}$

$f_{ss}=$ ____ $f_{tt} =$ ____

Solution:

$f(s, t) = 18(8s^2 + 5t^2)^{1/2}$

$$f_s = \frac{18}{2}(8s^2 + 5t^2)^{-1/2}(16s) = 144s(8s^2 + 5t^2)^{-1/2}$$

$$f_t = \frac{18}{2}(8s^2 + 5t^2)^{-1/2}(10t) = 90t(8s^2 + 5t^2)^{-1/2}$$

$$f_{ss} = 144s\left[-\frac{1}{2}(8s^2 + 5t^2)^{-3/2}(16s)\right] + (8s^2 + 5t^2)^{-1/2}(144)$$

$$= \frac{-1152s^2}{(8s^2 + 5t^2)^{3/2}} + \frac{144}{(8s^2 + 5t^2)^{1/2}}\frac{(8s^2 + 5t^2)}{(8s^2 + 5t^2)}$$

$$= \frac{720t^2}{(8s^2 + 5t^2)^{3/2}}$$

$$f_{tt} = 90t\left[-\frac{1}{2}(8s^2 + 5t^2)^{-3/2}(10t)\right] + (8s^2 + 5t^2)^{-1/2}(90)$$

$$= \frac{720t^2}{(8s^2 + 5t^2)^{3/2}}$$

$$f_{st} = \frac{\partial}{\partial t}(f_s) = 144s\left[-\frac{1}{2}(8s^2 + 5t^2)^{-3/2}(10t)\right]$$

$$= \frac{-720st}{(8s^2 + 5t^2)^{3/2}}$$

$$f_{ts} = \frac{\partial}{\partial s}(f_t) = 90t\left[-\frac{1}{2}(8s^2 + 5t^2)^{-3/2}(16s)\right]$$

$$= \frac{-720st}{(8s^2 + 5t^2)^{3/2}} = f_{st}$$

Correct answer is: $f_{ss} = \frac{720t^2}{(8s^2 + 5t^2)^{\frac{3}{2}}}$ $f_{tt} = \frac{720s^2}{(8s^2 + 5t^2)^{\frac{3}{2}}}$

11.

Given the function of three variables $f(x, y, z) = xy + xz + yz$, evaluate $f(1, -2, 5)$.

Your Answer: ____

Solution:

Substituting $x = 1$, $y = -2$, $z = 5$ into the formula for $f(x, y, z)$, we get

$f(1, -2, 5) = (1)(-2) + (1)(5) + (-2)(5) = -7$

Correct answer is: -7
