Version A

Lecture 28 Solutions

Solutions

1.

Find the maximum and minimum values of $e(x, y) = 5e^{xy}$ subject to $9x^2 + 9y^2 =$ 378.

maximum: ____ , minimum: ____

Solution:

$f(x, y) = 5e^{xy}$

$g(x, y) = 9x^2 + 9y^2 - 378 = 0$

$f_x = 5ye^{xy}, f_y = 5xe^{xy}, g_x = 18x$ and $g_y = 18y$

The three Lagrange equations are

$5ye^{xy} = 18\lambda x$

$5xe^{xy} = 18\lambda y$

$9x^2 + 9y^2 - 378 = 0$

Dividing the first two leads to $\frac{y}{x} = \frac{x}{y}$, or $x^2 = y^2$.

Substitute in $9x^2 + 9y^2 = 378$ to obtain

$x = \pm\sqrt{21}$ and $y = \pm\sqrt{21}$.

Now, $f(\sqrt{21}, -\sqrt{21}) = f(-\sqrt{21}, \sqrt{21}) = 5e^{-21}$ and

$f(\sqrt{21}, \sqrt{21}) = f(-\sqrt{21}, -\sqrt{21}) = 5e^{21}$.

So, the constrained maximum is $5e^{21}$ and the constrained minimum is $5e^{-21}$

Correct answer is: maximum:$5e^{21}$, minimum: 5

e^{21}