Version A

Solutions

1.

A manufacturer determines that when *x* hundred units of a particular commodity are produced, they can all be sold for a unit price given by the demand function $p = 26 - x$ dollars. What is the maximum revenue?

The manufacturer should expect maximum revenue of $ ____ .

Solution:

The revenue derived from producing *x* hundred units and selling them all at $26 - x$ dollars is $R(x) = x(26 - x)$ hundred dollars. Note that $R(x) \geq 0$ only for $0 \leq x \leq 26$.

The graph of the revenue function

$R(x) = x(26 - x) = -x^2 + 26x$

is a parabola that opens downward (since $A = -1 < 0$) and has its high point (vertex) at

$$x = -\frac{B}{2A} = \frac{-26}{2(-1)} = 13$$

Thus, revenue is maximized when $x = 13$ hundred units are produced, and the corresponding maximum revenue is

$R(13) = 13(26 - 13) = 169$

hundred dollars. The manufacturer should produce 1300 units and at that level of production, should expect a maximum revenue of $16900.

Note that we can also find the largest value of $R(x) = -x^2 + 26x$ by completing the square:

$R(x) = -x^2 + 26x = -(x^2 - 26x)$	*factor out* -1, *the coefficient of* x
$= -(x^2 - 26x + 169) + 169$	*complete the square inside parentheses by adding* $(-26/2)^2 = 169$
$= -(x - 13)^2 + 169$	

Thus, $R(13) = 0 + 169 = 169$ and if *c* is any number other than 13, then

$R(c) = -(c - 13)^2 + 169 < 169 \qquad since\ -(x - 13)^2 < 0$

so the maximum revenue is 169 when $x = 26$.

Correct answer is: 16900

2.

Find the slope of the line joining the points (−5, 1) and (3, −6).

Slope = ____

Solution:

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{-6 - 1}{3 - (-5)} = \frac{-7}{8}$$

The line is shown in the figure below.

Correct answer is: Slope = $-\frac{7}{8}$

3.

Write an equation for the line that passes through (−1, −4) and is perpendicular to the line 3*x* + 2*y* = 10.

y = ____

Solution:

By rewriting the equation $3x + 2y = 10$ in the slope-intercept form

$y = -\frac{3}{2}x + 5$, we see that the given line has slope $m_1 = -\frac{3}{2}$.

A perpendicular line has a slope of $m_1 = -1/m_2 = \frac{2}{3}$.

Given that the point (−1, −4) is on the line,

$$y + 4 = \frac{2}{3}(x + 1)$$

$$y = \frac{2}{3}x - \frac{10}{3}.$$

Correct answer is: $y = \frac{2}{3}x - \frac{10}{3}$

4.

Write an equation for the line that passes through (−9, 6) and is parallel to the line $6x + 5y = 5$.

y = ____

Solution:

By rewriting the equation $6x + 5y = 5$ in the slope-intercept form

$y = -\frac{6}{5}x + 1$, we see that the given line has slope, $m = -\frac{6}{5}$.

Since parallel lines have the same slope, $m = -\frac{6}{5}$ for the desired line.

Using the point (−9, 6) in the point-slope formula yields

$$y - 6 = -\frac{6}{5}(x + 9)$$

$$y = -\frac{6}{5}x + \frac{-24}{5}$$

Correct answer is: $y = -\frac{6}{5}x - \frac{24}{5}$
