Version A

Lecture 5 Practice Solutions

Solutions

1.

Find the one-sided limits $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$ from the given graph of f and determine whether $\lim_{x \to 2} f(x)$ exists.

A. $\lim_{x \to 2^-} f(x) = 3$, $\lim_{x \to 2^+} f(x) = 3$, and $\lim_{x \to 2} f(x) = 3$.

B. $\lim_{x \to 2^-} f(x) = 3$, $\lim_{x \to 2^+} f(x) = -2$, and $\lim_{x \to 2} f(x)$ does not exist.

C. $\lim_{x \to 2^-} f(x) = -2$, $\lim_{x \to 2^+} f(x) = 3$, and $\lim_{x \to 2} f(x)$ does not exist.

D. $\lim f(x) = -2$, $\lim f(x) = -2$, and $\lim f(x) = -2$.

$x \to 2^-$ $x \to 2^+$ $x \to 2$

Solution:

As x approaches 2 from the left, the curve approaches the point (2, −2) so $\lim_{x \to 2^-} f(x) = -2$.

From the right the curve approaches the point (2, 3) so $\lim_{x \to 2^+} f(x) = 3$.

Since the one-sided limits at $x = 2$ are not equal $\lim_{x \to 2} f(x)$ does not exist.

Correct answer is: C

2.

Find lim $\frac{x-4}{x-8}$ as x approaches 8 from the left.

$\lim_{x \to 8^-} \frac{x-4}{x-8} =$ ____

Solution:

Note that for $4 < x < 8$ the quantity

$$f(x) = \frac{x-4}{x-8}$$

is negative, so as x approaches 8 from the left, $f(x)$ *decreases* without bound. We denote this fact by writing

$$\lim_{x \to 4^-} \frac{x-4}{x-8} = -\infty$$

Correct answer is: $\lim_{x \to 8^-} \frac{x-4}{x-8} = -\infty$

3.

Find the indicated one-sided limits.

$$\lim_{x \to 2^-} f(x) \quad \text{and} \quad \lim_{x \to 2^+} f(x)$$

where $f(x) = \begin{cases} \dfrac{1}{4 - x} & \text{if } x < 2 \\ 3x^2 + 3x & \text{if } x \geq 2 \end{cases}$

$\lim_{x \to 2^-} f(x) =$ ____ ; $\lim_{x \to 2^+} f(x) =$ ____

Solution:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{1}{4 - x} = \frac{1}{4 - 2} = \frac{1}{2}$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3x^2 + 3x = 3(2)^2 + 3(2) = 18$$

Correct answer is: $\lim_{x \to 2^-} f(x) = \frac{1}{2}$; $\lim_{x \to 2^+} f(x) = 18$

4.

Is the rational function $f(x) = \dfrac{x + 10}{x - 10}$ continuous at $x = 11$?

A. The rational function is continuous at $x = 11$.

B. The rational function is discontinuous at $x = 11$.

Solution:

Note that $f(11) = \frac{11 + 10}{11 - 10} = 21.$

Since $\lim_{x \to 11} (x - 10) \neq 0$, you find that

$$\lim_{x \to 11} f(x) = \lim_{x \to 11} \frac{x + 10}{x - 10} = \frac{\lim_{x \to 11} (x + 10)}{\lim_{x \to 11} (x - 10)} = \frac{21}{1} = 21 = f(11)$$

as required for $f(x)$ to be continuous at $x = 11$.

Correct answer is: A

5.

Discuss the continuity of the following function:

$$g(x) = \frac{x^2 - 64}{x + 8}$$

- **A.** $g(x)$ **is continuous for all real numbers** x**.**
- **B.** $g(x)$ **is continuous except at** $x = 8$**.**
- **C.** $g(x)$ **is continuous except at** $x = -8$**.**
- **D.** $g(x)$ **is continuous except at** $x = 0$**.**

Solution:

The function is rational and is therefore continuous wherever it is defined (that is, wherever its denominator is not zero).

Since $x = -8$ is the only value of x for which $g(x)$ is undefined, $g(x)$ is continuous except at $x = -8$.

Continuous for $x \neq -8$

Correct answer is: C

6.

For what value of the constant A is the following function continuous for all real x?

$$f(x) = \begin{cases} Ax - 1 & \text{if } x < 1 \\ x^2 - 9x + 3 & \text{if } x \geq 1 \end{cases}$$

***f* is continuous for all *x* only when $A =$ ____ .**

Solution:

Since $Ax - 1$ and $x^2 - 9x + 3$ are both polynomials, it follows that $f(x)$ will be continuous everywhere except possibly at $x = 1$. Moreover, $f(x)$ approaches $A - 1$ as x approaches 1

from the left and approaches -5 as x approaches 1 from the right. Thus, for $\lim_{x \to 1} f(x)$ to exist, we must have $A - 1 = -5$ or $A = -4$, in which case

$$\lim_{x \to 1} f(x) = -5 = f(1)$$

This means that f is continuous for all x only when $A = -4$.

Correct answer is: -4

7.

Discuss the continuity of the function

$$f(x) = \frac{x + 1}{x - 3}$$

on the open interval $-1 < x < 3$ and on the closed interval $-1 \leq x \leq 3$.

A. **Discontinuous on the open interval $-1 < x < 3$ but not on the closed interval $-1 \leq x \leq 3$.**

B. **Continuous on the open interval $-1 < x < 3$ and on the closed interval $-1 \leq x \leq 3$.**

C. **Discontinuous on the open interval $-1 < x < 3$ and on the closed interval $-1 \leq x \leq 3$.**

D. **Continuous on the open interval $-1 < x < 3$ but not on the closed interval $-1 \leq x \leq 3$.**

Solution:

The rational function $f(x)$ is continuous for all x except $x = 3$. Therefore, it is continuous on the open interval $-1 < x < 3$ but not on the closed interval $-1 \leq x \leq 3$, since it is discontinuous at the endpoint 3 (where its denominator is zero).

The graph of f is shown below.

Correct answer is: D