Version A

Lecture 6 Practice

Solutions

1.

Compute the derivative of the given function and find the slope of the line that is tangent to its graph for the specified value of the independent variable.

$\boldsymbol{f(x) = 7x^2 - 7x + 12; x = 0}$

$f'(x) =$ ____ and the slope is $m =$ ____

Solution:

If $f(x) = 7x^2 - 7x + 12$, then $f(x + h) = 7(x + h)^2 - 7(x + h) + 12$.

The difference quotient (DQ) is

$$\frac{f(x+h) - f(x)}{h} = \frac{[7(x+h)^2 - 7(x+h) + 12]}{h} - \frac{[7x^2 - 7x + 12]}{h}$$

$$= \frac{14xh + 7(h)^2 - 7h}{h}$$

$$= 14x + 7h - 7$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 14x - 7.$$

The slope is $m = f'(0) = -7$.

Correct answer is: $f'(x) = 14x - 7$ and the slope is $m = -7$

2.

Compute the derivative of the given function and find the slope of the line that is tangent to its graph for the specified value of the independent variable.

$$g(t) = \frac{2}{t}\ ;\ t = \frac{1}{6}$$

$g'(t) =$ ____ and the slope is $m =$ ____

Solution:

If $g(t) = \frac{2}{t}$, then $g(t+h) = \frac{2}{t+h}$.

The difference quotient (DQ) is

$$\frac{g(t+h) - g(t)}{h} = \frac{\frac{2}{(t+h)} - \frac{2}{t}}{h}$$

$$= \frac{\frac{2}{(t+h)} - \frac{2}{t}}{h} \cdot \frac{t(t+h)}{t(t+h)}$$

$$= \frac{2t - 2(t+h)}{h(t)(t+h)}$$

$$= \frac{-2}{t(t+h)}$$

$$g'(t) = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} = -\frac{2}{t^2}.$$

The slope is $m = g'\left(\frac{1}{6}\right) = -72$.

Correct answer is: $g'(t) = -\frac{2}{t^2}$ and the slope is $m = -72$

3.

First compute the derivative of $f(x) = \sqrt{x}$, then use it to find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point where $x = 9$.

$y =$ ____

Solution:

The derivative of $y = \sqrt{x}$ with respect to x is given by

$$\frac{d}{dx}(\sqrt{x}) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}} \qquad \text{if } x > 0$$

When $x = 9$, the corresponding y coordinate on the graph of $f(x) = \sqrt{x}$ is $y = \sqrt{9} = 3$, so the point of tangency is $P(9, 3)$.

Since $f'(x) = \frac{1}{2\sqrt{x}}$, the slope of the tangent line to the graph of $f(x)$ at the point $P(9, 3)$ is given by

$$f'(x) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

and by substituting into the point-slope formula, we find that the equation of the tangent line at P is

$$y - 3 = \frac{1}{6}(x - 9)$$

or

$$y = \frac{1}{6}x + \frac{3}{2}$$

Correct answer is: $y = \frac{1}{6}x + \frac{3}{2}$

4.

Let $f(x) = 2x^2 - 6x$.

Find the average rate of change of $f(x)$ with respect to x as x changes from $x = 0$ to $x = \frac{7}{5}$.

$m =$ _____

Solution:

If $f(x) = 2x^2 - 6x$, then $f(0) = 0$, and

$$f(\tfrac{7}{5}) = 2(\tfrac{7}{5})^2 - 6(\tfrac{7}{5}) = \frac{-112}{25}$$

$$m_{\text{sec}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{-\frac{112}{25} - 0}{\frac{7}{5} - 0} = \frac{-16}{5}$$

Correct answer is: $m = -\frac{16}{5}$

5.

Differentiate the given function.

$$y = 14x^6$$

$\frac{dy}{dx}$ = ____

Solution:

$$\frac{d}{dx}(14x^6) = 14\frac{d}{dx}(x^6)$$

$$= 14(6x^5)$$

$$= 84x^5$$

Correct answer is: $\frac{dy}{dx} = 84x^5$

6.

Differentiate the polynomial $y = 2x^3 - 4x^2 + 13x - 1$.

$\frac{dy}{dx}$ = ____

Solution:

Differentiate this sum term by term to get

$$\frac{dy}{dx} = \frac{d}{dx}[2x^3] + \frac{d}{dx}[-4x^2] + \frac{d}{dx}[13x] + \frac{d}{dx}[-1]$$

$$= 6x^2 - 8x^1 + 13x^0 + 0 \qquad \textit{recall } x^0 = 1$$

$$= 6x^2 - 8x + 13$$

Correct answer is: $\frac{dy}{dx} = 6x^2 - 8x + 13$

7.

Differentiate the given function.

$$y = 5x^4 - 8x^{-9}$$

$\frac{dy}{dx} =$ ____

Solution:

$$\frac{d}{dx}[5x^4 - 8x^{-9}] = 5\frac{d}{dx}[x^4] - 8\frac{d}{dx}[x^{-9}]$$

$$= 5(4x^3) - 8(-9x^{-10})$$

$$= 20x^3 + 72x^{-10}$$

Correct answer is: $\frac{dy}{dx} = 20x^3 + 72x^{-10}$

8.

Differentiate the given function.

$$y = 85\sqrt{2x}$$

$\frac{dy}{dx} =$ ____

Solution:

$$y = 85\sqrt{2x} = 85\sqrt{2} \bullet x^{1/2}$$

$$\frac{dy}{dx} = 85\sqrt{2}\left(\frac{1}{2}x^{1/2-1}\right) = 85\sqrt{2}\left(\frac{1}{2}x^{-1/2}\right)$$

$$= 85\sqrt{2} \cdot \frac{1}{2x^{1/2}}$$

$$= \frac{85}{\sqrt{2}\, x^{1/2}}$$

$$\text{or} \quad \frac{85}{\sqrt{2x}}$$

Correct answer is: $\frac{dy}{dx} = \frac{85}{\sqrt{2x}}$
