Version A

Lecture 7 practice

Solutions

1.

Differentiate the product $P(x) = (x - 1)(8x - 11)$ by the product rule.

$P'(x) =$ ____

Solution:

By the product rule

$$P'(x) = (x-1)\frac{d}{dx}[8x-11] + (8x-11)\frac{d}{dx}[x-1]$$

$$= (x-1)(8) + (8x-11)(1) = 16x - 19$$

Correct answer is: $P'(x) = 16x - 19$

2.

Differentiate the quotient $Q(x) = \dfrac{x^2 - 3x + 1}{2x}$ by using the quotient rule.

A. $Q'(x) = \dfrac{2x}{4} - \dfrac{1}{2x^2}$

B. $Q'(x) = \dfrac{1}{2} + \dfrac{1}{2x^2}$

C. $Q'(x) = \frac{1}{2} - \frac{1}{2x^2}$

D. $Q'(x) = \frac{1}{2} - \frac{1}{2x}$

Solution:

By the quotient rule

$$Q'(x) = \frac{(2x)\frac{d}{dx}[x^2 - 3x + 1] - (x^2 - 3x + 1)\frac{d}{dx}[2x]}{(2x)^2}$$

$$= \frac{(2x)(2x - 3) - (x^2 - 3x + 1)(2)}{4x^2}$$

$$= \frac{2x^2 - 2}{4x^2}$$

$$= \frac{1}{2} - \frac{1}{2x^2}$$

Correct answer is: C

3.

Differentiate the given function.

$$f(t) = \frac{5t}{t^2 - 2}$$

A. $\frac{5t^2 + 10}{(t^2 - 2)^2}$

B. $\dfrac{5}{(t^2 - 2)}$

C. $\dfrac{-5t^2 - 10}{(t^2 - 2)^2}$

D. $-\dfrac{5}{(t^2 - 2)}$

Solution:

$$f'(t) = \frac{(t^2 - 2)\frac{d}{dt}(5t) - 5t\frac{d}{dt}(t^2 - 2)}{(t^2 - 2)^2}$$

$$= \frac{(t^2 - 2)(5) - (5t)(2t)}{(t^2 - 2)^2}$$

$$= \frac{-5t^2 - 10}{(t^2 - 2)^2}$$

Correct answer is: C

4.

Differentiate the given function.

$$y = \frac{t^2 + 10}{5 - t^2}$$

$$\frac{dy}{dt} = \text{\_\_\_\_}$$

Solution:

$$\frac{dy}{dt} = \frac{(5 - t^2)(2t) - (t^2 + 10)(-2t)}{(5 - t^2)^2}$$

$$= \frac{30t}{(5 - t^2)^2}$$

Correct answer is: $\frac{dy}{dt} = \frac{30t}{(5 - t^2)^2}$

5.

The gross domestic product (GDP) of a certain country was $N(t) = t^2 + 9t + 94$ billion dollars t years after 1980. At what percentage rate was the GDP changing with respect to time in 2002? Round your answer to the nearest integer.

The percentage rate of change of the GDP in 2002 was ____ % per year.

Solution:

The rate of change of the GDP in the derivative $N'(t) = 2t + 9$.

The rate of change in 2002 was $N'(22) = 2(22) + 9 = 53$ billion dollars per year.

The percentage rate of change of the GDP in 2002 was

$$100\frac{N'(22)}{N(22)} = 100\frac{53}{776} \approx 7\% \text{ per year}$$

Correct answer is: 7

6.

Suppose the demand q and the price p for a certain commodity are related by the linear equation $q = 600 - 6p$ (for $0 \leq p \leq 100$). Express the elasticity of demand as a function of p.

$E(p) =$ ____

Solution:

The elasticity of demand is

$$E(p) = \frac{p}{q}\frac{dq}{dp} = \frac{p}{q}(-6) = \frac{-6p}{600-6p} = \frac{-p}{100-p}$$

Correct answer is: $E(p) = \dfrac{-p}{100-p}$

7.

True or false?

The demand is inelastic at the indicated price p.

$$D(p) = -9.5p + 37;\ p = 2$$

True **False**

Solution:

The elasticity of demand is

$$E(p) = \frac{p}{q}\frac{dq}{dp} = \frac{p}{-9.5p+37}(-9.5) = \frac{-9.5p}{-9.5p+37}$$

When $p = 2$, the elasticity of demand is

$$E(2) = \frac{-9.5(2)}{-9.5(2)+37} \approx -1.1$$

and

$$|E(2)| \approx 1.1 > 1$$

Thus, the demand is elastic and the statement is false.

Correct answer is: **False**

8.

Suppose the demand q and the price p for a certain commodity are related by the linear equation $q = 160 - 2p$ (for $0 \le p \le 80$). At what price is the elasticity of demand equal to −1?

$p =$ ____

Solution:

The elasticity of demand is

$$E(p) = \frac{p}{q}\frac{dq}{dp} = \frac{p}{q}(-2) = \frac{-2p}{160 - 2p} = \frac{-p}{80 - p}$$

The elasticity of demand will be equal to −1 when

$$-1 = \frac{-p}{80 - p} \qquad 80 - p = p \qquad 2p = 80 \qquad \text{or} \qquad p = 40$$

At this price, a 1% increase in price will result in a decrease in demand of approximately the same percentage.

Correct answer is: 40

9.

Differentiate the given function.

$$y = \frac{\mathbf{12x - 3}}{\mathbf{17x + 7}}$$

$$\frac{dy}{dx} = \text{____}$$

Solution:

$$\frac{dy}{dx} = \frac{(17x + 7)\dfrac{d}{dx}(12x - 3) - (12x - 3)\dfrac{d}{dx}(17x + 7)}{(17x + 7)^2}$$

$$= \frac{12(17x + 7) - 17(12x - 3)}{(17x + 7)^2}$$

$$= \frac{135}{(17x + 7)^2}$$

Correct answer is: $\frac{dy}{dx} = \frac{135}{(\{17x\} + 7)^2}$
