SOLUTION #6 (9 AM)

Solution 1. $f'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$, *and notice only* $x = 1$ *in* $[\frac{1}{2}, 3]$. *(1pt)*
$f''(x) = (-1)(-2)\frac{1}{x^3} = \frac{2}{x^3}$, $f''(1) = 2 > 0 \Rightarrow f(x)$ *takes its absolute minimum at* $x = 1$, $f(1) = 1 + 1 = 2$. *(1pt)*
$f(\frac{1}{2}) = \frac{1}{2} + 2 = 2.5$ *(1pt)*
$f(3) = 3 + \frac{1}{3} = 3\frac{1}{3} > 2.5$, *so* $f(x)$ *takes its absolute maximum at* $x = 3$, $f(3) = 3\frac{1}{3}$. *(1pt)*

Solution 2. (1) $f'(x) = -\frac{5}{3}x^{\frac{2}{3}} = 0 \Rightarrow x = 0$, *so the function has only one critical point* $(0, 0)$. *(2pts)*

(2) $f''(x) = -\frac{5}{3} \cdot \frac{2}{3} \cdot x^{-\frac{1}{3}}$ *is not defined at* $x = 0$, *so the second derivative test does not work. (2pts)*

(3) *The only critical point* $x = 0$ *splits the real line into* $(-\infty, 0), (0, +\infty)$.
Pick $x = -1$ *in* $(-\infty, 0)$, $f'(-1) = -\frac{5}{3}(-1)^{\frac{2}{3}} = -\frac{5}{3} < 0$
Pick $x = 1$ *in* $(0, +\infty)$, $f'(1) = -\frac{5}{3}(1)^{\frac{2}{3}} = -\frac{5}{3} < 0$
Hence by the first derivative test, $(0, 0)$ *is not a relative extremum. (2pts)*