

SOLUTION #8 (8 AM)

Solution 1. (1) $Q(0) = A(1 - e^0) = A(1 - 1) = A \cdot 0 = 0$

(2) $Q'(t) = A(-1 \cdot (-k)e^{-kt}) = kAe^{-kt}$

(3) $\lim_{t \rightarrow \infty} Q(t) = \lim_{t \rightarrow \infty} A(1 - e^{-kt}) = A(1 - \lim_{t \rightarrow \infty} \frac{1}{e^{kt}}) = A \cdot (1 - 0) = A$

(4) $A = 10$

$$A(15) = 5 \implies 10(1 - e^{-k \cdot 15}) = 5 \implies 1 - e^{-15kt} = \frac{5}{10} = \frac{1}{2}$$
$$\implies e^{-15kt} = \frac{1}{2} \implies -15k = \ln\left(\frac{1}{2}\right) = -\ln 2 \implies k = \frac{\ln 2}{15}$$

Solution 2. (1) $f'(x) = 2xe^{-x} + x^2 \cdot (-1)e^{-x} = (2x - x^2)e^{-x}$

(2) $g'(x) = 2 \ln(2x) \cdot (\ln(2x))' = 2 \ln(2x) \frac{2}{2x} = \frac{2 \ln(2x)}{x}$